Individual Decision-Making to Commit a Crime:  
Early Models  
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1. Introduction

In the paper we present an introduction to a part of the economics of crime - individual decision-making to commit a crime. This theory started appearing in the literature in the 1960s and especially in the 1970s in an attempt to describe and predict human behavior concerning issues such as offence and crime. Several models have been developed and we discuss them in turn, especially the studies of Becker (1968), Ehrlich (1973) and Heineke (1978).

Because the success of offence is naturally uncertain in these models, we deal with the maximization of von Neumann - Morgenstern expected utility function. Moreover, we consider the offence risky and therefore encounter portfolio models where the individual allocates the wealth among legal and illegal activities. It is also reasonable to assume that these activities have a time dimension. As a result, individuals allocate time to legal and illegal activity. Determining the level of crime committed it is possible to derive the corresponding supply curve. The early models of economics of crime are similar to the models of portfolio choice and of the supply of labor.

We do not include in our survey the theory of optimal law enforcement. This theory extends the theory of individual decision-making in several aspects e.g. in the analysis of how law enforcement agents should deal with the unproductive behavior of those

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who commit crime in order to maximize social welfare. For survey see Polinsky and Shavell (2000). Also modern dynamic theories on tax evasion are not the aim of this paper. Survey is in Slemrod and Yithzaki (2002).

In section 2 we present portfolio models on individual decision-making to commit crime. We discuss the model of time allocation in section 3. We conclude in section 4. Two appendices on the derivation of some results of Heineke’s model I. follow.

2. Individual Decision-Making to Commit a Crime as a Rational Act

We assume that economic agents are rational, in other words, they fulfill the following assumptions in their decision-making: completeness, transitivity, reflexivity, non-satiation, continuity, and strict convexity. These assumptions are sufficient condition for rational behavior. Due to our assumptions we realize that our analysis of decision-making to commit a crime is most suitable to apply to areas such as the level of pollution that firms generate, robbery, tax evasion or traffic offence. The applicability of these models to emotional crime – or more generally to the case of unstable preferences - is in our opinion ambiguous.

Recently, there has been plenty of theoretical and empirical literature on the problem that individuals do not behave fully rationally. But even if we find that individual decision-making is not always rational (which can be actually to, a large extent, caused by the optimization of the amount of information to be evaluated or by learning), we can think of it as a useful approximation of reality.

2.1 Model of Dividing an Individual’s Wealth between Legal and Illegal Activities

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1 For survey of various attitudes towards rationality see Hamlin (1986), p.1-57. Short survey of early odels of economics of crime is provided by Milanovic (1999), see pages 5-11.
In this section we present the model of dividing an individual’s wealth between legal and illegal activities. We assume that individuals maximize well-defined von Neumann - Morgernstern expected utility function. As a result, the theory of individual decision-making to commit a crime is the special case of the general theory of rational behavior under uncertainty. Individual’s wealth consists of an exogenous (initial) wealth and additional income obtained from legal and illegal activities. Legal activities are without risk. Individuals are able to estimate the probability of all possible opportunities and monetize all the gains and losses.

The models of rational behavior to commit crime in formal economic terms appear in the literature from 1960s, starting with Becker’s (1968) pioneering article Crime and Punishment. Although, the first attempts to study crime from the economic point of view date back to the 18th and 19th centuries (see Eide (1994, p.48)).

2.1.1 Becker’s Model

Becker (1968) primarily focuses on minimizing social loss in income from the crime and not only on the individual decision-making. His article is a seminal work on the economics of crime in formal economic terms and is the basis for all further research.

Becker (1968) argues what is the optimal policy in order to combat crime and how it relates to the means of the punishment, public expenditures, probability of convictions, and private enforcement of law. Thus optimal policies to fight crime are a part of an optimal allocation of resources. We discuss especially the parts relevant to our survey.

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2 Distinction between initial and additional income is introduced by Brown and Reynolds (1973).
3 If we challenge the strict monetization, the results of the early models become more ambiguous. See Block and Heineke (1975).
4 However, there were also some unpublished manuscripts at Columbia University before 1968 such as Smigel-Leibowitz (1965) or Ehrlich (1967) focused on empirical assessment of crime activities in the U.S. Becker (1996, p. 143) also writes: “I began to think about crime in the 1960s after driving to Columbia University for an oral examination of a student in economic theory”.

3
Another of his results is that crime is socially undesirable since the potential offenders spend their time planning and implementing the crime, in other words on unproductive activities, which in turn causes only a violent income redistribution in society. In the modern literature this behavior is called rent-seeking.

Becker (1968) also discusses the economics of crime from the view of individual and stresses the rationality and the liability of the actions. The individual compares the benefits and the costs of committing crime (or offence). The crime is committed only if \( g > pf \), where \( g \) is the gain from the crime and the term \( pf \) is expected punishment (\( p \) stands for probability of punishment and \( f \) for fine). Moreover, from the point of view of the enforcer it is rational to increase fines and lower the probability of punishment so that the expected punishment would not change. The only limitation is the offender’s income. However, if this is true, why does the substitution between fines and probability of punishment have weak empirical grounds?

Polinsky and Shavell (1979) refine Becker’s results by considering the attitude towards risk. This together with the recent model of Garoupa (2001) explains why we do not observe this substitution in reality. Garoupa (2001) shows that substitability between the probability of punishment and fine holds only if the expected punishment is close to the gain from crime. Otherwise, the relationship can be complementary.

Becker (1968) derives that the greater elasticity of the change in the probability of punishment than the elasticity of response to the change in fines implies that offenders are marginally risk-lovers. Brown and Reynolds (1973) generalize Becker’s model risk implications about entering into illegal activities by showing relevant elasticities implies nothing about risk attitudes. Later on, there is vast theoretical literature refining the results of Becker’s model, such as Ehrlich (1973) or Polinsky and Shavell (1979).

\[ [5] \text{See Polinsky and Shavell (1991) for a formal model where differences in income are introduced.} \]
2.1.2 Heineke Model I

Heineke (1978) adds interesting aspects to Becker’s model and to the models on tax evasion\(^6\) from the early 1970s. He enriches both the models of portfolio choice and previous models of the economics of crime.

In the model the following notation is used:

- \(W^0\) - initial wealth;
- \(x\) - amount of \(W^0\) spent on illegal activity, \(0 \leq x \leq 1\), \(x \leq W^0\);
- \(g(x, a)\) - gain from offence, \(\partial g/\partial x > 0\), \(\partial g/\partial a > 0\), \(g(0, a) = 0\);
- \(f(x, \beta)\) - fine from offence, if detected\(^7\) \(\partial f/\partial x > 0\), \(\partial f/\partial \beta > 0\), \(f(0, \beta) = 0\);
- \(a\) - a shift parameter for gains (magnitude of the gains);
- \(\beta\) – a shift parameter for losses (severity of the losses);
- \(p\) - probability of detection;
- \(W^S\) - wealth when being successful (no fine imposed);
- \(W^U\) - wealth when being unsuccessful (fine is imposed on the offender);
- \(u(W)\) - von Neumann - Morgenstern utility, \(\partial u/\partial W > 0\);\(^8\)

The individuals maximize expected utility (EU):

\[
E[U(W)] = (1 - p)u(W^S) + pu(W^U) \tag{1}
\]

\[
\mu = \frac{\partial EU}{\partial x} = (1 - p)\frac{\partial u}{\partial x}(W^S)\frac{\partial g}{\partial x} + p\frac{\partial u}{\partial x}(W^U)(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}) = 0 \tag{2}
\]

It is clear from the first-order condition that such optimum can exist, since the first term in the equation (2) is positive and the second term is negative.\(^9\) It is natural to expect that \(\partial g/\partial x < \partial f/\partial x\), that is marginal gain should be lower than marginal fine, otherwise all the individuals would commit as much crime as possible. On the other hand, the condition for committing additional crime is:

\(^6\) For contemporary survey of the literature on tax evasion see Slemrod and Yitzhaki (2002), for the tax evasion models preceding Heineke’s model I. see Allingham and Sandmo (1972), Kolm (1973) and Singh (1973).
\(^7\) We assume for simplicity that detection equals to punishment. That is there are no costs to impose fine.
\(^8\) If \(\partial u/\partial W \partial W > 0\), then the individual is risk lover, if \(\partial u/\partial W \partial W < 0\) than the individual is risk averse, if \(\partial u/\partial W \partial W = 0\), than the individual is risk neutral.
\[
\frac{\partial g}{\partial x} > p \frac{\partial f}{\partial x}
\]  

(3),

this means the additional (marginal) gain has to be higher than the expected additional fine.

To make sure that we found the maximum of expected utility function; the second derivative of the equation (1) must be negative.

\[
\frac{\partial EU}{\partial x} = (1 - p) \frac{\partial u}{\partial x} (W^s) \frac{\partial g}{\partial x} + (1 - p) \frac{\partial u}{\partial x} (W^s) \frac{\partial g}{\partial x} + p \frac{\partial u}{\partial x} (W^u)
\]

\[
\frac{\partial g}{\partial x} + p \frac{\partial u}{\partial x} (W^u) \frac{\partial g}{\partial x} - p \frac{\partial u}{\partial x} (W^u) \frac{\partial g}{\partial x} - p \frac{\partial u}{\partial x} (W^u) \frac{\partial g}{\partial x} < 0
\]  

(4).

We can see that the inequality (4) can be fulfilled independently on the attitude towards risk.

After the exclusion of potential corner solutions (x = 0, x = 1), we can find the individual’s response to the change of parameters. In other words, how much maximizing would an individual dedicate to illegal activities when there is a change in his/her wealth, the probability of detection, the size of gain and the severity of losses? Under the constant absolute risk aversion we obtain\(^{10}\)

\[
\frac{\partial \mu}{\partial W} = 0
\]  

(5)

This means change in the wealth will not affect the incidence of individual’s illegal activities.

Under the decreasing absolute risk we have

\[
\frac{\partial \mu}{\partial W} > 0
\]  

(6).

As a result, more wealth will bring more crime. Wolpin (1978) shows a rather surprising conclusion from the equation (6). Illegal activity decreases with increasing unemployment, since higher unemployment rate implies lower income. Notice an interesting feature that in this case we cannot explain with the perception quoted by Eide (1994, p.56):” … misery creates a situation where crime is a rational

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\(^9\)Because \(p\) and \(\frac{\partial u}{\partial W} > 0\) and \(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x} < 0\).

\(^{10}\) See Appendix 1 for derivation of the results how optimal value of x changes with the change of wealth.
reaction.” Social norms as Eide (1994) argues can be appropriate for explaining this problem.

On the other hand, using the results of Polinsky and Shavell (1999) and Garoupa (2001) we may derive how to explain equation (6) by the rational behavior at variance with Eide (1994). Low wealth means low opportunity costs and low probability being fined by a substantial amount. This change in the other parameters may in turn explain why equation (6) stands under rational behavior, too.

Under the increasing absolute risk aversion we end up with

\[ \frac{\partial \mu}{\partial W} < 0 \] (7).

This means higher wealth contributes to lower illegal activity.

Next, we can derive under some reasonable assumptions the following outcomes: \(^{11}\)

\[ \frac{\partial \mu}{\partial \beta} < 0, \frac{\partial \mu}{\partial p} < 0 \text{ and } \frac{\partial \mu}{\partial a} > 0 \] (8).

The results from equation (8) are as follows: more severe punishment causes less crime, as well as a higher probability of detection. On the other hand, higher gains from crime make individual more likely to be involved in illegal activities.

3. Model of Time Allocation

A possible limitation of the portfolio models (Heineke model I in the last section) is that individual has not only to decide about the optimal level of crime, but to consider time as an important variable influencing the decision-making, too. As a result, we encounter the models where time will be allocated between legal and illegal activities. We redefine the problem of the individual’s maximizing of the expected utility as follows:

\[ E[U(W)] = (1-p)u(W^S) + pu(W^L) \] (9),

where

\[ W^S = W^0 + L(t_e, \delta) + G(t_e, \chi) \]

\(^{11}\) See Appendix 2 for derivation.
Individual maximizes the expected utility with respect to $t_i$ (time spent on illegal activity) and $t_L$ (time spent on legal activity). Because the algebra is very similar to the portfolio model, we do not present the derivations.\(^\text{12}\) All the assumptions from the previous model apply, too.

Now, there are two possibilities how to deal with the model. First, as Heineke (1978) discusses, the simple model without restrictions on the values of $t_i$ and $t_L$. Second introduces the restriction on a sum of the value $t_i$ and $t_L$. That means $t_i + t_L = t$, where $t$ means time and is a positive real number. The second model is developed in Ehrlich (1973).

### 3.1 Heineke’s model II

An individual will maximize with respect to $t_L$ and $t_i$ the following:

$$E[U(W)] = (1 - p)u(W^S) + pu(W^U)$$  \hspace{1cm} (10)

The resulting first-order conditions are:

$$\frac{\partial EU}{\partial t_L} = (1 - p)\frac{\partial u}{\partial t_L} (W^S) \frac{\partial L}{\partial t_L} + p \frac{\partial u}{\partial t_L} (W^U) \frac{\partial L}{\partial t_L} = 0$$  \hspace{1cm} (11)

$$\frac{\partial EU}{\partial t_i} = (1 - p)\frac{\partial u}{\partial t_i} (W^S) \frac{\partial L}{\partial t_i} + p \frac{\partial u}{\partial t_i} (W^U) \left( \frac{\partial G}{\partial t_i} - \frac{\partial F}{\partial t_i} \right) = 0$$  \hspace{1cm} (12)

\(^\text{12}\) See Ehrlich (1973), Heineke (1978) or Eide (1994) for survey.
The optimal value of $t_L$ depends on benefits and costs arising from legal activities and is independent of $p$, $W^0$, $\chi$ and $\Psi$ (probability of detection, initial wealth, the magnitude of gains and the size of the imposed fine respectively). But increase in the wealth causes that individual will allocate more of his/her time to legal activities. When $\partial L/\partial t_L = 0$ (the marginal benefits reach zero), an individual will exert oneself to illegal activities. It is possible to derive the following inequality:

$$\partial t_i/\partial p < 0$$

(13).

Time involved in illegal activity decreases as the probability of punishment increases.

The outcome of change in the severity of punishment, the magnitude of gains, gains from legal activities and wealth increase on time allocated on illegal activities (sign of these partial derivatives $\partial t_i/\partial \Psi$, $\partial t_i/\partial \chi$, $\partial t_i/\partial \delta$ and $\partial t_i/\partial W$ respectively) depends on the attitude towards risk.

In the case of constant absolute risk aversion we get:

$$\partial t_i/\partial \chi > 0, \partial t_i/\partial \Psi < 0, \partial t_i/\partial \delta = 0, \partial t_i/\partial W = 0$$

(14).

This means, if there is increase in the gains from the illegal activities, individual will concentrate more on it. If the punishment from illegal activities is more severe, then everybody decreases its illegal activities.

In the case of increasing absolute risk aversion we obtain:

$$\partial t_i/\partial \chi > 0, \partial t_i/\partial \Psi < 0, \partial t_i/\partial \delta < 0, \partial t_i/\partial W > 0$$

(15).

The results in the equation (15) show that individual devotes less time to illegal activities when higher severity of punishment is present and when there are better legal opportunities. Increase in the wealth and the magnitude of gains makes the individual tend more to the illegal activities. The results for the decreasing absolute risk aversion are the following:

$$\partial t_i/\partial \chi > 0, \partial t_i/\partial \Psi > 0, \partial t_i/\partial \delta > 0, \partial t_i/\partial W > 0$$

(16).

Higher potential gains as well as higher wealth from illegal activities cause higher crime. Better legal opportunities do not have negative influence on a incidence of

13 See Eide (1994) for mathematical derivation, the technique is roughly the same as in the portfolio model, we do not present it.
crime committed. The consequence of various severity of punishment on the level of 
crime (the sign of $\partial t_i / \partial \Psi$) is indeterminate.

If there is a large independence between legal and illegal activities, the model yields 
the same result as portfolio Heineke model I.

### 3.2 Ehrlich’s Model

We discuss the model of Ehrlich (1973) in this section. In this model we assume that 
the total time is fixed and has to be divided between legal and illegal activities. As a 
result, the more time is spent on legal activities, the less may be spent on illegal 
activities and vice versa. We can rewrite the wealth depending on success in the 
following manner:

$$W^S = W^0 + L(t-t_i, \delta) + G(t_i, \chi)$$

$$W^U = W^0 + L(t-t_i, \delta) + G(t_i, \chi) - F(t_i, \Psi)$$

The first-order condition of the optimization of the allocation of time is: \(^{14}\)

$$\frac{\partial EU}{\partial t_i} = (1-p) \frac{\partial u}{\partial t_i} (W^U) \left( \frac{\partial L}{\partial t_i} + \frac{\partial G}{\partial t_i} - \frac{\partial F}{\partial t_i} \right) + (1-p) \frac{\partial u}{\partial t_i} (W^S) \left( \frac{\partial G}{\partial t_i} - \frac{\partial L}{\partial t_i} \right) = 0 \tag{17}$$

Rearranging leads to:

$$\frac{\partial G}{\partial t_i} - \frac{\partial L}{\partial t_i} = \frac{p}{1-p} \frac{\partial u}{\partial t_i} (W^U) \tag{18}$$

The term on the left describes the individual’s indifference curve. After the 
differentiation of equation (10) and setting $dE[U(W)] = 0$, we have:

$$\frac{dW^S}{dW^U} = \frac{p}{1-p} \frac{\partial u}{\partial t_i} (W^U) \tag{19}$$

Also can be found out, that left term in the equation (18) is the marginal rate of 
substitution between $W^S$ and $W^U$ given the overall time constant. As $t_i$ increases, it

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\(^{14}\) The second-order condition assures the maximum, see Ehrlich (1973, p.527).
will produce transformation curve, where $W^S$ is substituted for $W^U$ according to the following equation:

\[
\frac{\partial G}{\partial t_i} \frac{\partial L}{\partial t_i} = \frac{dW^S}{dW^U}
\]

Equations (19) and (20) determine the optimal time allocated on legal and illegal activities (point B in Figure 1). If all the time was devoted to the illegal activities, than the individual’s optimum would become point C. In addition, if there is only legal activity, individual will end up in the point A. Notice also that certainty line must lie at the angle of $45^\circ$, since we assume that legal earnings are certain and also gains and losses $[G(t, \chi) \text{ and } F(t, \Psi)]$ are zero and then $W^S = W^U$. Naturally, transformation curve’s domain is $\langle W^0 + G(t) - F(t), W^0 + L(t) \rangle$ and its relevant range is $\langle W^0 + L(t), W^0 + G(t) \rangle$ (see figure 1).

**Figure 1 – Determination of Activity Mix**

![Diagram showing determination of activity mix with points A, B, and C, and lines for transformation curve and certainty line.](image)

*Source: Ehrlich (1973)*

It can be also shown that if risk-neutral person would choose point B in the figure 2, than an optimum for risk-lover must be left to the point B on the transformation curve and to the right for a risk-averse person.
It is clear that individual participates in illegal activities, if his/her expected utility increased. To state this formally:

\[
\frac{\partial EU}{\partial t} > 0 \quad (21).
\]

Concretely, when there is no illegal activity, \( W^U = W^S \) and thus \( u^U = u^S \). Putting equation (19) and (21) together, after rearranging we obtain:

\[
p \frac{p \partial G}{\partial t} - \frac{p \partial L}{\partial t} > p \frac{\partial F}{\partial t} \quad (22).
\]

The marginal differential gain must exceed the expected punishment in order to crime to occur.

Ehrlich’s analysis goes beyond Becker (1968) in some aspects, since the model explores not only costs, but benefits, too. Similarly, Ehrlich’s model can predict not only the direction of the changes as was the case of Heineke’s model II, but also the magnitude of the legal and illegal activities. Clearly, Ehrlich’s model also provides the arguments for punishing repeated offenders more severely.

4. Conclusion

Some types of criminal activity are largely explicable by the rational decision-making of individuals. As we could see in our survey of the early models, namely the portfolio and time allocation model, unsurprisingly they bring very similar results. In all cases regardless of attitudes towards risk, the higher probability of punishment lowers the efforts dedicated to committing an offence. The results of the change in the severity of punishment, the magnitude of gains from illegal activities and income are indeterminate and depend on the attitude towards risk.

However, the positive models of the individual decision-making to commit a crime do not analyze many aspects of the criminal activities such as how enforcement agents should behave in order to maximize social welfare and the unproductive behavior of those, which plan and commit crimes. The models consider only individual decision-making and ignore interactive decision-making, as well as more than one time period

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decision-making, which is crucial in analyzing e.g. marginal deterrence. It may be derived from the early models that the optimal punishment is maximal. This conclusion is drawn because of the nature of the models (comparative static models). The idea of maximal punishment was later refuted.

On the other hand, the models of individual decision-making provide a useful introduction and a guideline to the economics of crime and highlight the fact that it is always the rational individual who is at center in analyzing the consequences of decision-making.

We conclude with the quotation by Ehrlich (1973, p.527): “Recidivism is not necessarily the result of an offender's myopia, erratic behavior, or lack of self control, but may rather be the result of choice dictated by opportunities.”

**Literature**


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15 But not all the models are comparative-static models, e.g. Allingham, Sandmo (1972) present also dynamic model of decision-making of tax evader, where several time periods are considered in the


decision-making.
Appendix 1

Derivation of the sign \( \partial \mu / \partial W \) [equations (5), (6) and (7)]:

From the first-order condition (equation (2)) we have \( \mu = 0 \) and differentiating we get

\[
d\mu = \frac{\partial \mu}{\partial W} dW + \frac{\partial \mu}{\partial x} dx = 0
\]

Rearranging we have: \[16\]

\[
\frac{dx}{dW} = -\frac{\partial \mu / \partial W}{\partial \mu / \partial x}
\]

Since \( \partial \mu / \partial x \) is negative (second-order condition), it is enough to explore the sign of \( \partial \mu / \partial W \) in order to determine the sign of \( dx/dW \). Taking the derivative of \( \mu \) with respect to \( W \) we get:

\[
\frac{\partial \mu}{\partial W} = (1 - p) \frac{\partial \mu}{\partial \tilde{x} W} (W^5) \frac{\partial g}{\partial x} + p \frac{\partial \mu}{\partial \tilde{x} W} (W^U) (\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x})
\]

From the first-order condition (equation (2)) we have

\[
(1 - p) \frac{\partial f}{\partial x} = -p \frac{\partial u / \partial x(W^U)}{\partial u / \partial x(W^5)} (\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x})
\]

Putting together we obtain

\[
\frac{\partial \mu}{\partial W} = -p \frac{\partial u / \partial x(W^U)}{\partial u / \partial x(W^5)} (\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}) \frac{\partial u}{\partial \tilde{x} W} (W^5) + p \frac{\partial u}{\partial \tilde{x} W} (W^U) (\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x})
\]

Rearranging leads to

\[
\frac{\partial \mu}{\partial W} = -p \frac{\partial g}{\partial x} (\frac{\partial f}{\partial x}) \frac{\partial u / \partial x(W^U)}{\partial u / \partial x(W^5)} \left[ -\frac{\partial u / \partial x W^5(W^5)}{\partial u / \partial x(W^5)} - (\frac{\partial u / \partial \tilde{x} W(W^5)}{\partial u / \partial x(W^5)}) \right]
\]

where

\[
r_x(W^5) = -\frac{\partial u / \partial \tilde{x} W(W^5)}{\partial u / \partial x(W^5)}
\]

and

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\[16\] The more proper way would be to solve the derivation by implicit function as it is in Eide (1994), however the results are the same.
Finally, we have
\[
\frac{\partial u}{\partial W} = -p\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\right)\frac{\partial u}{\partial x(W^U)}\left[r_A(W^S) - r_A(W^U)\right]
\]
Under the constant absolute risk aversion (CARA) \( r_A(W^S) = r_A(W^U) \) and that means \( \partial u/\partial W = 0 \). Under the decreasing absolute risk aversion (DARA) \( r_A(W^S) < r_A(W^U) \) and that means \( \partial u/\partial W > 0 \). Under the increasing absolute risk aversion (IARA) \( r_A(W^S) > r_A(W^U) \) and that means \( \partial u/\partial W < 0 \). We obtain these results, because \( p \) and \( \partial u/\partial W(W^U) \) are positive and the term \( (\partial g/\partial x - \partial f/\partial x) \) is negative. If we recapitulate:
CARA \( \Rightarrow \partial u/\partial W = 0 \), DARA \( \Rightarrow \partial u/\partial W > 0 \) and IARA \( \Rightarrow \partial u/\partial W < 0 \).

Appendix 2
Derivation of \( \partial u/\partial \beta < 0 \), \( \partial u/\partial p < 0 \) and \( \partial u/\partial a > 0 \) from the equation (8).

We employ the same technique as in Appendix 1.
First, we show how the severity of fines effects the criminal activity and how the result depend on the attitude on risk:
\[
du = \frac{\partial u}{\partial \beta} \, d\beta + \frac{\partial u}{\partial x} \, dx = 0
\]
Rearranging we get:
\[
\frac{dx}{\partial \beta} = -\frac{\partial u/\partial \beta}{\partial u/\partial x}
\]
and then
\[
\frac{\partial u}{\partial \beta} = p\left[ \frac{\partial u}{\partial x}(W^U)\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\right)\frac{\partial u}{\partial x}(W^U)(-\frac{\partial f}{\partial \beta}) + \frac{\partial u}{\partial x}(W^U)\frac{\partial f}{\partial \beta}\right]
\]
For a risk-averse and risk-neutral individual \( \mu_{\beta} \) is obviously always negative. For risk-loving individual the result is uncertain, but if the following inequality holds
\[
\frac{\partial u}{\partial x}(W^U)\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\right)\frac{\partial u}{\partial x}(W^U)(-\frac{\partial f}{\partial \beta}) < \frac{\partial u}{\partial x}(W^U)\frac{\partial f}{\partial \beta}
\]
then \( \partial u/\partial \beta < 0 \), too.
Second, we show that the increase in the probability of detection always decreases the criminal activity of the individual:

\[ \frac{\partial \mu}{\partial p} = -\frac{\partial u}{\partial x} (W^s) \frac{\partial g}{\partial x} + \frac{\partial u}{\partial x} (W^s) \left( \frac{\partial g}{\partial x} - \frac{\partial f}{\partial x} \right) < 0 \]

Third, we show how the magnitude of the gain effects illegal activity. After some algebra we get:

\[ \frac{\partial \mu}{\partial \alpha} = \frac{\partial u}{\partial W} \frac{\partial g}{\partial x} + \frac{\partial g}{\partial x \partial \alpha} \left( -\frac{\partial EU / \partial W}{\partial \mu / \partial x} \right) \]

Since we already derived CARA \( \Rightarrow \frac{\partial \mu}{\partial W} = 0 \), DARA \( \Rightarrow \frac{\partial \mu}{\partial W} > 0 \) and IARA \( \Rightarrow \frac{\partial \mu}{\partial W} < 0 \). \( \frac{\partial \mu}{\partial x} < 0 \) is a sufficient condition for maximizing the expected utility and \( \frac{\partial g}{\partial \alpha} > 0 \). Then we only have to assume the marginal gain must increase, that is \( \frac{\partial g}{\partial x \partial \alpha} > 0 \), and can derive the sign of \( \frac{dx}{da} \).

As a result, for decreasing absolute risk aversion and constant absolute risk aversion we obtain \( \frac{\partial \mu}{\partial \alpha} > 0 \). The bigger the gain the more illegal activities will be present. The outcome under increasing absolute risk aversion is indeterminate.