Durables, Nondurables, Down Payments and Consumption Excesses*

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Abstract

We examine a model that generalizes the standard buffer-stock model of saving to accommodate durables, nondurables, and a collateralized constraint, with and without adjustment costs in the durables market. Since there is no known analytical solution to the model, we solve it numerically. We find that nondurable consumption becomes more volatile relative to income as down payment requirements decrease at the individual and the aggregate level. Moreover, for plausible parameter values, the model can explain the excess smoothness and excess sensitivity observed in U.S. aggregate data. This result follows from a gradual adjustment of consumption to permanent income shocks when agents attempt to spread out the burden of down payments over time.

KEYWORDS: Buffer stock, Consumption, Durable Goods, Incomplete Markets, Computational Economics.
JEL classificiations: E21 - Consumption; Saving, C36 - Computational Techniques, C61 - Optimization Techniques; Programming Models; Dynamic Analysis.

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1 Introduction

Private consumption is the most important component of aggregate demand. Nevertheless, modelling consumption behavior is still a challenge to the economics profession. Understanding consumption is essential for constructing meaningful macroeconometric forecasting models, as well as for evaluating the effects of key fiscal and monetary policies.

One of the lasting contributions to modern day consumer theory is the idea that consumption is determined by the expected value of lifetime resources or permanent income, known as the life cycle-permanent income hypothesis model (LC-PIH).\(^1\) The modern-day specification of this fundamental concept involves an intertemporal choice model with quadratic preferences, stochastic labor income, no borrowing restrictions, and perfect foresight. In spite of its intuitive appeal, several influential papers reveal discrepancies between this model’s predictions and the aggregate data. On the one hand, Hall’s (1978) paper surprised the profession by demonstrating that under simple assumptions, consumption is a martingale; i.e., a regression of period \(t\) consumption growth on any variable known at period \(t - 1\) should return an estimate of zero. Regressions using aggregate data, however, consistently return an estimate significantly larger than zero when current growth in consumption is regressed on lagged aggregate income growth—a phenomenon known as “excess sensitivity” (of current consumption to lagged income).\(^2\) On the other hand, the LC-PIH model also predicts that if the income process exhibits high persistence, current consumption should respond strongly to unanticipated innovations. Empirical work using aggregate data consistently finds a small reaction of consumption to current income shocks—a phenomenon known as “excess smoothness”.\(^3\)

The buffer-stock model of saving, pioneered by Deaton (1991) and Carroll (1997), is a promising candidate for replacing the LC-PIH model as the benchmark model of consumer behavior. This model, while still preserving many of the insights stemming from rational forward-looking behavior, assumes individuals cannot (or endogenously will not) borrow and allows consumers to be more prudent and less patient than in Hall (1978). Over the last decade, a large body of literature has shown that the buffer-stock model can explain several aspects of household spending decisions. However, at the aggregate level, the implications of the buffer-stock model are not as well explored, and in some cases,

\(^1\)This idea was pioneered by Modigliani and Brumberg (1954) and Friedman (1957).
\(^2\)For studies of excess sensitivity, see Flavin (1981), Blinder and Deaton (1985), and Campbell and Deaton (1989).
\(^3\)Campbell and Mankiw (1987) and Cochrane (1988), among others, document that innovations to GNP are highly persistent. Building on the results in Hansen and Sargent (1981), excess smoothness has been documented by Deaton (1987), Campbell and Deaton (1989), and Gali (1991).
not fully satisfactory. Ludvigson and Michaelides (2001) show—in a careful and explicit aggregation of the buffer-stock model—that the model cannot generate robust excesses. Instead, the authors rely on incomplete information as in Pischke (1995) to generate some excesses. Similarly, Michaelides (2001) needs habit formation to generate the excesses observed in the aggregate data.

We believe one main shortcoming of most consumption models, including the standard buffer-stock model, is that they traditionally focus solely on the study of nondurable consumption.\footnote{Notable exceptions are Caballero (1993) and Eberly (1994) who focus on durables, and Chah, Ramey, and Starr (1995), Alessie, Devereux, and Weber (1997), Carroll and Dunn (1997), Dunn (1998) and Flavin and Nakagawa (2004), who consider models with both durables and nondurables.} In this paper, we present a generalized buffer-stock model with durables and nondurables that can explain the excess sensitivity and excess smoothness of nondurable consumption observed in the aggregate data without the need for incomplete information.

Introducing durable goods into this framework is not straightforward and can be done in several different ways. In our specification, we assume individuals derive utility from consumption of a nondurable good and from the services provided by a durable good. Moreover, the durable good can act as collateral for credit purchases. In particular, the durable can be financed minus a down payment, and durable equity loans, with a certain maximum loan-to-value ratio, are available to consumers. Collateralized constraints of this type impose distortions on the allocation of consumption across time and across goods, even with a utility function separable in both goods.\footnote{Even without collateralized liquidity constraints, the interactions between durable and nondurable goods are interesting. Browning and Crossley (1997) show that individuals who face limited borrowing alternatives smooth out fluctuations in income by postponing the replacement of small durables.} We also take into account the fact that the market for durables may be characterized by important transaction costs.

We consider non-convex costs of adjustment as in Grossman and Laroque (1990), which generate large and infrequent adjustments, in a $(S, s)$ rule fashion.\footnote{See Attanasio (1998) for more references and an insightful discussion of models with lumpy adjustment.} In all other respects, our model is identical to the classic buffer-stock framework.

The use of a collateralized constraint should not be controversial. First, according to the Federal Reserve Board’s 1998 Survey of Consumer Finances (SCF), collateral borrowing, mainly obtained to purchase housing and automobiles, is the principal type of borrowing undertaken by households.\footnote{In the 1998 SCF, 92 percent of all available credit to households is for the purchase of houses and automobiles (collateral credit). Moreover, the average ratio of collateral credit to total debt across households is roughly 79 percent.} Second, considering this constraint allows us to study an extra motive for savings: saving for down payments. Down payments represent a large financial burden for most households. For example, according to the annual survey of home buyers...
Who’s Buying Homes in America?, households must save, on average, for two and a half years to buy their first home. Moreover, by changing down payment requirements, our model can be used to study certain implications of financial liberalization.

It is important to acknowledge that a very similar formulation of the problem without adjustment costs has been explored by Chah, Ramey, and Starr (1995) and Alessie, Devereux, and Weber (1997). However, the focus of these papers is empirical. Carroll and Dunn (1997) present a similar model with adjustment costs but focus on the role of unemployment expectations on consumption spending decisions. To our knowledge, this particular version of the model has not yet been solved. We show that with a combination of the right numerical dynamic programming techniques, the curse of dimensionality and the complications that the illiquidity of the durable poses can be overcome, and reasonable parameterizations of the model can be solved accurately. In particular, we use Euler equation iteration for a version of the model with no adjustment costs in the durable market, and a finite state approximation method for the version with adjustment costs. Both techniques are briefly discussed and compared.

After solving the model, we characterize the optimal consumption rules for an individual consumer under different down payment regimes. We then simulate individual and aggregate consumption series—calculated through explicit aggregation—and study the implications of different down payments for consumption patterns. Finally, we explore whether a plausible parametrization of the model can account for the excess sensitivity and excess smoothness observed in the aggregate data for nondurable consumption.

At the individual level, we find that nondurable consumption is smoother relative to income when down payment requirements are high for two different reasons. First, when income is transitorily low, a buffer-stock consumer on occasion liquidates the equity accumulated in the durable to prop up his nondurable consumption. Since higher required down payments translate into higher levels of equity, nondurable consumption becomes smoother. Second, when an individual experiences a positive permanent income shock, he chooses not to fully adjust his consumption due to the desire to spread out the cost of accumulating a down payment. The implications for the durable are slightly more complex and are thoroughly discussed throughout the paper.

At the aggregate level, nondurable consumption is also smoother for higher down payments. The result follows from a gradual adjustment of consumption due to the desire

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8See Chicago Title and Trust (1995). For first-time buyers, the majority of the required down payment comes from savings (74.8 percent). For repeat buyers, savings are supplemented by the proceeds from the existing house sale (52.2 percent from savings and 31.7 percent from the sale of the previous home).
to spread out the cost of the down payment, not the higher equity levels associated with higher down payments. At the aggregate level, all individual-specific shocks cancel out and what remains is the effect of aggregate shocks, which we model as permanent. The sluggish response of consumption to changes in income can generate robust excesses for reasonable parameter values, especially in the presence of transaction costs.

In other words, with lower down payments, consumption becomes more volatile relative to income and excess sensitivity weakens. This implication is consistent with the international empirical evidence in Japelli and Pagano (1989) who find that down payment requirements are highly correlated with excess sensitivity. Furthermore, Bacchetta and Gerlach (1997) find that excess sensitivity varies over time with a clear tendency to decline in U.S. aggregate data. More recently, Peersman and Pozzi (2004) show an inverse relation between excess sensitivity coefficients and measures of financial liberalization in U.S. aggregate data. Both findings are also consistent with our model. A further interesting implication of the model is that average wealth holdings decrease with decreases in down payments or transaction costs.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 describes the solution method, the calibration, as well as the optimal policy functions for nondurable and durable consumption. In Section 4, we discuss the implications of the model on consumption patterns for both an individual household and for the aggregate. Section 5 provides concluding remarks.

2 The Basic Model

The consumer’s problem is to maximize the present discounted value of expected utility from consumption of a nondurable good, $C_t$, and from the service flow provided by a durable good, $K_t$, where $t$ denotes time. We assume that time is discrete and agents face an infinite horizon. $\beta < 1$ is the discount factor.\footnote{This prediction is not inconsistent with the evidence that consumption volatility and output volatility decrease with financial development as documented in Denizer, Iyigun, and Owen (2002). Our model predicts increases in the volatility of consumption relative to the volatility of income. If financial liberalization reduces the volatility of income, the absolute volatility of consumption decreases in our model as well.}

\footnote{Note the simplifying assumption that an agent’s service flow from the durable is proportional to the durable stock. Furthermore, we set the constant of proportionality equal to one. This simplification is used by many others. See, for example, Mankiw (1982) and Chah, Ramey, and Starr (1995). A slightly more realistic setup would express utility as a function of the service flows derived from the durable stock. These services could be affected by, among other things, frequency of use.}

4
\[
\max_{(C_t, K_t)} V = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, K_t) \right\}.
\]

(1)

The instantaneous utility function is assumed to be separable in both goods and is of the CRRA type:

\[
U(C_t, K_t) = \frac{C_t^{1-\rho}}{1-\rho} + \varphi \frac{K_t^{1-\rho}}{1-\rho},
\]

(2)

where \(\varphi\) is a preference parameter.\(^{11}\) Note that \(\rho > 0\) implies that the agent is risk-averse and has a precautionary motive for saving.

This maximization is subject to both a budget constraint and a wealth constraint. Assume there is one riskless financial asset, \(A_t\). \(R\) is the interest factor paid on it. In period \(t\), an agent holds past financial assets gross of interest, \(RA_{t-1}\), and receives \(Y_t\) units of income. In the same period, the agent chooses nondurable consumption, \(C_t\), and net investment on the durable, \((K_t - \psi K_{t-1})\), where \(\psi\) is the depreciation factor. Moreover, the agent may be subject to an adjustment cost, \(\zeta(K_t, K_{t-1})\), when changing the durable stock. The budget constraint between two successive periods is given by:

\[
A_t = RA_{t-1} + Y_t - C_t - (K_t - \psi K_{t-1}) - \zeta(K_t, K_{t-1}).
\]

(3)

Labor income is assumed to be exogenous to the agent and stochastic, and is the only source of uncertainty in the model. We assume, as in Ludvigson and Michaelides (2001) and similar to Carroll (1997), that labor income, \(Y_t\), is the product of permanent income, \(P_t\), and an idiosyncratic transitory shock, \(T_t\): \(Y_t = P_tT_t\). In turn, permanent income is \(P_t = G_tP_{t-1}N_t\). \(G_t\) can be thought of as the growth in permanent income attributable to aggregate productivity growth in the economy which is common to all agents. \(N_t\) is a permanent idiosyncratic shock. We assume \(\ln G_t, \ln T_t,\) and \(\ln N_t\) are independent and identically normally distributed with means \(\mu_G, \mu_T,\) and \(\mu_N\), and variances \(\sigma^2_G, \sigma^2_T,\) and \(\sigma^2_N\) respectively. This income specification is particularly useful since it allows for consumers to share in general growth while the variance of their income can be calibrated to be dominated by idiosyncratic permanent or transitory components. The specification implies that the growth rate of individual labor income follows an MA(1) process, \(\Delta \ln Y_t = \ln G_t + \ln N_t + \ln T_t - \ln T_{t-1}\), which is consistent with the microeconomic evidence.\(^{12}\) By

\[\text{We follow Bernanke (1984) who studies the joint behavior of the consumption of durable and non-durable goods and finds that separability was a good approximation. With regards to prices, we assume that } P^C_t/P^K_t = 1, \forall t.\]

\[\text{See the discussion in Ludvigson and Michaelides (2001) on how empirical studies such as MaCurdy (1982), Abowd and Card (1989), and Pischke (1995) find that an MA(1) in the growth rate of income is }\]
the law of large numbers, aggregate income, $\bar{Y}_t$, follows the process, $\Delta \ln \bar{Y}_t = \ln G_t + 0.5 \sigma^2_N$, where $\bar{Y}_t = 1/n \sum_{i=1}^{n} Y_{it}$, $i = 1, \ldots, n$.

A very important aspect of the model is the collateralized constraint imposed on the agent:

$$A_t + (1 - \theta)K_t \geq 0,$$

with $\theta \in [0, 1]$. This constraint implies that an individual’s borrowing limit is a fraction $(1 - \theta)$ of the durable stock. The constraint summarizes several commonly observed aspects of collateral lending. A household can only finance a fraction $(1 - \theta)$ of durable purchases. In other words, it must satisfy a down payment requirement $\theta$. The constraint also implies that when a household owns a durable good, it can obtain a durable equity loan with a maximum loan-to-value ratio $(1 - \theta)$. In summary, at any point in time, an agent is only required to keep an accumulated durable equity, $\theta K_t$. Note that total wealth, $A_t + K_t$, can be divided into a required down payment (required equity or required wealth), $\theta K_t$, and the wealth held in excess of the required down payment or voluntary equity, $Q_t \equiv A_t + (1 - \theta)K_t$. Also, the consumer can increase nondurable consumption by decreasing either voluntary or required equity. However, accessing required equity implies changing the durable stock, which may be costly if adjustment costs are present. Finally, this collateralized constraint does not imply a fixed borrowing limit but a limit that varies with the durable stock and $\theta$.\(^{13}\)

3 Solving the Model

A closed-form solution of the model does not exist and we must rely on computational methods to solve the consumer’s problem. This section presents our computational strategy as well as some qualitative implications of the solution. We first solve the model with no adjustment costs to understand the role of durability and the collateralized constraint. We then add non-convex adjustment costs to incorporate irreversibility and infrequent changes in durable purchases.

\(^{13}\)The standard-buffer stock model can be easily extended to allow for a fixed borrowing limit. Ludvigson (1999) studies a buffer-stock model of saving with time varying liquidity-constraints, where credit varies stochastically with income.
3.1 No Adjustment Costs

Euler equation iteration has been the traditional approach for solving microeconomic dynamic stochastic optimization problems with nondurable consumption only. We generalize the algorithm in Carroll (1997) and Deaton (1991) to accommodate multiple goods and the collateralized constraint considered here. We reformulate the model to facilitate the implementation of this numerical technique.

Reformulating the Model

Define cash-on-hand, $X_t$, as $X_t ≡ RA_{t-1} + ψK_{t-1} + Y_t$. Note that durable wealth can be lumped together with financial resources and labor income because we assume for now that there are no costs of adjusting the durable stock. The budget constraint becomes $A_t = X_t - C_t - K_t$ and the collateralized constraint $C_t + θK_t ≤ X_t$. Combining the definition of cash-on-hand and the budget constraint, we can write an expression for the evolution of cash-on-hand: $X_{t+1} = R(X_t - C_t) + (ψ - R)K_t + Y_{t+1}$. The first-order conditions of the problem are:

$$U_C^t = \beta R E_t[U_{C}^{t+1}] + \lambda_t, \quad (5)$$

$$U_K^t = \beta(R - ψ) E_t[U_{C}^{t+1}] + θ \lambda_t, \quad (6)$$

$$λ_t(X_t - C_t - θK_t) = 0. \quad (7)$$

Equation (5) states that the marginal utility of nondurable consumption in period $t$ must equal the discounted expected marginal utility of nondurable consumption in period $t+1$, plus the shadow price of the constraint. Analogously, equation (6) states that the marginal utility of durable consumption in period $t$ must be equal to the expected marginal utility of nondurable consumption in period $t+1$ discounted by $β(R - ψ)$, plus $θ$ times the shadow price of the constraint. Note the difference in the discount factor from the above equation—$βR$ versus $β(R - ψ)$—because of the durability of $K$. Also, $λ_t$ is multiplied by $θ$ to reflect the fact that only a down payment is required as payment for the durable in period $t$.

Equations (5) and (6) are intertemporal conditions. Solving for $βE_t[U_{C}^{t+1}]$ in equation (5) and plugging it into equation (6), we obtain an equation for the intratemporal relationship between $C_t$ and $K_t$:

$$U_K^t = \frac{R - ψ}{R} U_C^t + \left(θ - \frac{R - ψ}{R}\right) λ_t. \quad (8)$$
When the liquidity constraint is not binding, \( \lambda_t = 0 \). In this particular case, given our utility function, equation (8) implies:

\[
\frac{C_t}{K_t} = \varphi^{-\frac{1}{\rho}} \left( \frac{R - \psi}{R} \right)^{\frac{1}{\rho}} = \Omega. \tag{9}
\]

This is the optimal relationship between \( C_t \) and \( K_t \) accounting for durability.\(^{14}\) \((R - \psi)/R\) is known in the literature as the user cost of the durable. This cost represents the single-period cost, or rental equivalent cost of one durable unit. It is affected by the depreciation factor and the interest rate.\(^{15}\) When the agent is not constrained, the trade-off between \( C_t \) and \( K_t \) is fully captured by the user cost and the preference parameter. For constrained agents, other factors come into play. If \( K_t \) is poor collateral (\( \theta \) is higher than the user cost), constrained agents let durable consumption fall temporarily and vice versa. Note that when \( \theta = (R - \psi)/R \), the trade-off between \( C_t \) and \( K_t \) is determined only by the user cost, even if the constraint is binding. This is a particularly useful benchmark case since the constraint does not impose any distortions in the intratemporal allocation between the two goods.\(^{16}\)

In order to deal with the nonstationarity of income, we normalize all variables by permanent income, \( P_t \), as proposed by Carroll (1997). Lower-case variables denote upper-case counterparts divided by permanent income. The Euler-Lagrange equations can be rewritten as follows. When the agent is not constrained:

\[
\beta R E_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1} \left( R(x_t - c_t) + \left( \frac{\psi - R}{\Omega} \right) c_t \right) + T_{t+1} \right] \right)^{-\rho} \right\} - c_t^{-\rho} = 0. \tag{10}
\]

\(^{14}\)If \( \varphi = 1 \) and \( \psi \) is 0 (i.e. the durable depreciates completely after one period), \( U_t = U_t^k_t \). That is, the agent would choose to consume the same amounts of both goods \( (C_t = K_t) \). If \( \psi > 0 \), \( C_t < K_t \).

\(^{15}\)Depreciation erodes the agent’s investment in the durable and effectively increases the cost. The interest rate also increases the user cost as it reflects the opportunity cost of investing in the durable: a dollar invested in the durable could have returned \( R - 1 \) dollars if invested in financial assets. We ignore other factors, such as capital gains and losses on the durable. A more general specification for the user cost would be \((P_t^K R - P_{t+1}^K \psi)/(P_t^K R)\).

\(^{16}\)Note also that equations (5) and (8) imply that:

\[
R \beta E_t [U_C^{t+1}] - U_C^t = \left( \theta - \frac{R - \psi}{R} \right)^{-1} \left( U_K^t - \frac{R - \psi}{R} U_C^t \right). \]

The change in marginal utility between two successive periods is not white noise. This interesting implication of the model was used by Chah, Ramey, and Starr (1995) to empirically study the excess sensitivity of consumption.
When the agent is constrained:

\[
\beta[\psi - R(1 - \theta)] E_t \left\{ \left( G_{t+1} N_{t+1} \right)^{1 - \rho} \left( c_{t+1} \left[ \left( G_{t+1} N_{t+1} \right)^{-1} \left[ \psi - R(1 - \theta) \right] \frac{x_{t+1} - c_t}{\theta} + T_{t+1} \right] \right)^{-\rho} \right\} \\
- \theta c_t^{\rho} + \varphi \left( \frac{x_t - c_t}{\theta} \right)^{1 - \rho} = 0.
\]

Equations (10) and (11) can be solved to obtain a policy function for normalized non-durable consumption as a function of the only state variable, normalized cash-on-hand, \( c(x) \). Once we find the policy function for nondurable consumption, the policy function for durable consumption, \( k(x) \), can be calculated by using the intratemporal relationship between the two goods. Appendix A.1 presents further details, including convergence conditions.

**The Policy Functions**

We now describe the shape of the optimal consumption functions for normalized non-durable and durable consumption. In order to compare our findings with the previous literature, we start by adding up \( c_t \) to \( \theta k_t \). The policy function for this variable is depicted in Figure 1, panel A. Similar to Deaton (1991), there is a unique \( x^*(\theta) \) such that:

\[
c + \theta k = \begin{cases} 
  x, & x \leq x^*(\theta), \\
  < x, & x > x^*(\theta).
\end{cases}
\]

If normalized cash-on-hand this period, \( x_t \), is below a threshold level, \( x^*(\theta) \), the agent is constrained and all resources are exhausted after paying for nondurable consumption and the down payment requirement. In other words, no normalized voluntary equity is carried over to the next period, \( q_t = a_t + (1 - \theta) k_t = 0 \). If cash-on-hand is higher than \( x^*(\theta) \), some voluntary equity is accumulated, \( q_t > 0 \). Note that the higher \( \theta \) is, the higher the required level of equity an agent must keep. As a result, the threshold \( x^*(\theta) \) is an increasing function of \( \theta \) (see Figure 1, panel B).

[Figure 1]

How are resources are allocated between the two goods? Propositions 1 and 2 summarize our findings. Proofs are presented in Appendix A.3.
Proposition 1

When an agent is not constrained, \( x > x^*(\theta) \), \( c(x) = \frac{\Omega}{\Omega + \theta} (x - q) \), and \( k(x) = \frac{1}{\Omega + \theta} (x - q) \), regardless of the value of \( \theta \).

Proposition 1 states that when the agent is not constrained and once he has made the decision regarding how much voluntary equity to bring to the next period, the agent spends fixed proportions of the remaining cash-on-hand between the two goods.

Proposition 2

When an agent is constrained, \( x \leq x^*(\theta) \),

(A) If no down payment is required, \( \theta = 0 \),
\[
c(x) = x, \quad \text{and} \quad k(x) = \frac{1}{\Omega} x^*(\theta).
\]

(B) If the down payment parameter is lower than the user cost, \( \theta < \frac{R - \psi}{R} \), \( c(x) \) is a convex function of \( x \), and \( k(x) \) is a concave function of \( x \).

(C) If the down payment parameter is equal to the user cost, \( \theta = \frac{R - \psi}{R} \), \( c(x) \) and \( k(x) \) are linear functions of \( x \). In particular:
\[
c(x) = \frac{\Omega}{\Omega + \theta} x \quad \text{and} \quad k(x) = \frac{1}{\Omega + \theta} x.
\]

(D) If the down payment parameter is higher than the user cost, \( \frac{R - \psi}{R} < \theta \leq 1 \), \( c(x) \) is a concave function of \( x \), and \( k(x) \) is a convex function of \( x \).

Proposition 2 describes, for the case when the agent is constrained, the shapes of the policy functions which depend on the credit conditions characterized by the relationship between the down payment parameter and the user cost.

[Figure 2]

Figure 2 illustrates the propositions by depicting the policy rules for the four parameter regions described in Proposition 2. There are important differences in the consumption functions. For a constrained agent, the policy function for the durable good becomes “flatter” as the down payment requirement gets lower. The opposite is true for nondurable consumption. This implies that for a level of normalized cash-on-hand such that the agent is constrained in all regimes, the marginal propensity to consume the nondurable good out
of cash-on-hand is higher the lower $\theta$ (in fact, it is exactly one when $\theta = 0$). The shapes suggest, therefore, higher nondurable volatility and lower durable volatility for regimes with lower down payments but do not prove it. First, $x$ is endogenous, and second, as $\theta$ gets higher, agents are more likely to be constrained for a given level of $x$. Whether or not nondurable (durable) consumption indeed becomes more (less) volatile relative to income with decreasing down payments needs to be verified through simulation.

### 3.2 Adjustment costs

After gaining some understanding of the role of the collateralized constraint, we incorporate transaction costs in the durables market. Durables are typically purchased in large lumpy increments and changed only infrequently. Bar-Ilan and Blinder (1992), Caballero (1993), Bertola and Caballero (1990), and Eberly (1994) argue that optimal consumption rules for durables can be described as following an $(S,s)$ rule. When the stock of a durable good falls below some lower bound $s$, a purchase is made and the stock is readjusted to a target size $S$. As long as the stock of the durable good remains above the trigger point $s$, no action is taken. Non-convex costs of adjustment generate $(S,s)$ patterns and thus we choose a non-convex cost specification in our model. In particular we use a similar specification to Grossman and Laroque (1990):

$$\zeta(K_t, K_{t-1}) = \phi d \psi K_{t-1},$$

where $\phi$ is the adjustment cost parameter and $d$ is a dummy variable which takes on the value of zero when there is no investment, $K_t - \psi K_{t-1} = 0$, and one otherwise. This adjustment cost can be seen as a proportional loss in the selling price of the agent’s prior holdings of the durable stock. This loss in price can be attributable to any type of cost incurred upon sale, such as the payment of taxes, a sales commission, or an imperfection in the resale market for the durable.\(^{17}\) Note that once the agent has decided to adjust his durable holdings, the adjustment cost is fixed from his perspective; the cost is proportional to the inherited level of the durable stock, $\psi K_{t-1}$.\(^{18}\) In this formulation, the transaction cost does not diminish in importance as households become wealthier, as with a purely

\(^{17}\)See Lam (1989) for an analysis of the aggregate implications of the time series properties of durable expenditure when the irreversibility of incremental adjustment of the durable is due to resale market imperfections.

\(^{18}\)In this specification the adjustment cost is paid by the seller. Alternatively, we could divide the cost between buyer and seller: $\zeta(K_t, K_{t-1}) = \phi_1 d \psi K_{t-1} + \phi_2 d K_t$. Then, the effective payment paid by a consumer when purchasing the durable would be $\theta + \phi_2$. In order to keep the effects of down payments separate from the effects of adjustment costs, we choose the first specification.
fixed cost. This specification also implies that incremental adjustments do not occur (i.e. the agent must sell his entire existing stock upon adjustment).\textsuperscript{19}

Technically, adding adjustment costs is not a trivial modification. We cannot use Euler equation iteration to solve the model because of non-differentiability issues. Thus, we apply a different numerical dynamic programming technique, a finite state approximation method. In order to apply this technique, we must use an alternative reformulation of the model in terms of voluntary equity and the durable stock.

Given the adjustment cost, financial assets, $A_t$, evolve according to:

$$A_t = RA_{t-1} - (K_t - \psi (1 - d\phi)K_{t-1}) + Y_t - C_t. \quad (12)$$

The evolution of voluntary equity is given by:

$$Q_t = A_t + (1 - \theta)K_t$$

$$= RA_{t-1} - (K_t - \psi (1 - d\phi)K_{t-1}) + Y_t - C_t + (1 - \theta)K_t$$

$$+ R(1 - \theta)K_{t-1} - R(1 - \theta)K_{t-1}$$

$$= RQ_{t-1} + [\psi (1 - d\phi) - R(1 - \theta)]K_{t-1} - \theta K_t + Y_t - C_t. \quad (13)$$

The constraint becomes $Q_t \geq 0, \forall t$. In order to deal with the nonstationarity of income, we normalize all variables by permanent income. Then, we use the homogeneity of degree $(1 - \rho)$ property of the utility function to write the Bellman equation of the model as:

$$V(q_{t-1}, k_{t-1}) =$$

$$\beta E_{t-1} \left\{ \left( (G_t N_t) \right)^{1-\rho} \max_{q_t, k_t: q_t \geq 0} \left[ U \left( (G_t N_t) \right)^{-1} \left\{ Rq_{t-1} + [\psi (1 - d\phi) - R(1 - \theta)]k_{t-1} \right\} \right] - \theta k_t + V_t - q_t, k_t \right\} + V(q_t, k_t). \quad (14)$$

The solution technique consists of specifying and solving a finite-state problem that approximates the continuous one presented above. Note that under this formulation, the control variables ($q_t, k_t$) are also next period’s states. These continuous variables can be approximated by finite discrete sets. Moreover, the specification allows for a straightforward incorporation of the liquidity constraint and the adjustment cost. The discretely

\textsuperscript{19}This is more reasonable if $K$ represents a single durable good (i.e. a house) than if $K$ represents composite good.
approximated problem is solved using value function iteration combined with an acceleration technique, modified policy function iteration, as explained in Appendix A.2.

Adding adjustment costs obviously changes the policy functions for durables. Also, the role of durables as a substitute for a liquid buffer-stock of saving is diminished since selling durables to recover required equity is costly. In order to fully understand the effect that down payment requirements and transaction costs have on consumption behavior, we solve both versions of the model numerically and calculate several consumption statistics for both nondurable and durable goods under different down payment regimes (different $\theta$s), with and without adjustment costs. A numerical solution requires appropriate calibration of the model’s parameters, which we now describe.

3.3 Calibration

We use an annual horizon as most microeconomic evidence for the parameters that we must calibrate comes from studies of annual data. We set the relative risk aversion coefficient to 2, $\rho = 2$. The rate of time preference is 0.05—$\beta = 1/1.05$— and the net real interest rate equals 2 percent, $R = 1.02$. The income shocks parameters are as follows: $\mu_G = 0.02$, $\mu_N = \mu_T = 0$, $\sigma_G = 0.025$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. All values are similar to Ludvigson and Michaelides (2001), whose results we compare to ours.\footnote{In fact, these parameter values correspond to one of Ludvigson and Michaelides’s (2001) worst performing scenarios for the standard buffer-stock model of saving. Results are robust to small variations of the parameters, which we do not tabulate here for brevity.} The adjustment cost parameter, $\phi$, is 0 in the non-adjustment cost case and 5 percent in the adjustment cost case.

$\psi$, the depreciation factor, is set to 0.915, implying an annual depreciation rate of 8.5 percent. We obtain this number by combining data from the National Income and Product Accounts (NIPA) and the Fixed Assets and Consumer Durable Goods Accounts (FACD) from the Bureau of Economic Analysis for the years 1959–2001. We interpret durables, $K$, in a comprehensive manner as the sum of residential stocks and all consumer durable goods. Accordingly, investment on durables, $I$, is calculated as expenditure on consumer durables plus residential private domestic investment. We assume the U.S. is in a steady state and calculate the real, average ratio of investment on durables to the durable stock, which determines the depreciation rate: $1 - \psi = I/K$.\footnote{In the steady state, $\Delta K = I - (1 - \psi)K = 0$ which implies $1 - \psi = I/K$.}

We need to calibrate one last parameter, $\varphi$, the preference parameter in the utility function. We proceed as follows. First, we find the ratio of real nondurable consumption
to the durable stock \((C/K)\) using NIPA and FACD, which is 0.36.\(^{22}\) We know that when an agent is not constrained:

\[
\frac{C}{K} = \left(\frac{R - \psi}{R}\right)^{\frac{1}{\rho}} \varphi^{-\frac{1}{\rho}}.
\]

Given the values of \(\rho, R, \psi\) and \(C/K\), we obtain \(\varphi = 0.795\). We use this number for our individual consumption simulations and let the \(C/K\) ratio vary accordingly to illustrate the effects of a changing \(\theta\) on the ratio. For the aggregate consumption simulations, \(\varphi\) is adjusted to keep the ratio \(C/K\) constant and equal to 0.36 under the different down payment regimes.

In our simulations, the down payment parameter, \(\theta\), is set free for two reasons. First, our durable good is a composite of very different commodities: houses, cars, furniture, etc. \(\theta\) is the down payment parameter and \((1 - \theta)\) is the maximum loan-to-value ratio for durable equity loans. Not only are down payments likely to be different for the different categories, but while home equity loans are widely available, other durable equity loans at favorable rates are not as common. Second, certain aspects of the collateralized constraint we consider in this paper deviate from financial contracts written in reality. Mainly, in order to keep the model tractable, the down payment parameter is the same for all consumers and the borrowing rate is not a function of \(\theta\).\(^{23}\) Therefore, it is not obvious what the right value for \(\theta\) should be. Since for our sample period, houses represent 82 percent of the total durable stock in FADC data, we could argue then for values close to down payments for houses. According to the Federal Housing Finance Board (FHFB), the average down payment for the period 1963–2001 is 25 percent. In fact, less than 33 percent of homeowners put down less than 20 percent. However, we anticipate that in order to quantitatively account for the excess sensitivity and excess smoothness of consumption in aggregate data, we require a sufficient wedge between the user cost and the down payment parameter. Given our parameter choices, the user cost of the durable, \((R - \psi)/R\), is roughly 10 percent and we need down payments higher than this value to match the aggregate excesses.\(^{24}\)

\(^{22}\)C is defined as the sum of nondurable consumption plus services minus housing, and \(K\) as the sum of the private residential stock plus the stock of consumer durables. We keep shoes and clothing within the nondurable category since in this framework it is not appropriate to model them as durables. The results of the paper do not change significantly if these are ignored or treated as durables.

\(^{23}\)In addition to the collateral requirement, lenders impose several additional criteria to reduce the likelihood of default. For housing, some lenders require that the mortgage payment does not exceed some percentage of current income. Another standard condition requires the loan-to-value ratio (LTV) to be below a certain threshold. Otherwise, the borrower faces higher marginal borrowing costs, including a higher interest rate and the purchase of mortgage insurance.

\(^{24}\)In this paper, we abstract from house price appreciation which would reduce the user cost. If this
We use the parameters above to compute individual optimal policy rules for normalized nondurable and durable consumption. Next, we generate labor income shocks from the assumed distribution of idiosyncratic and aggregate shocks for 200 periods. Given the optimal policy rules and the simulated income realizations, we calculate nondurable and durable consumption (for that number of periods) for several individuals. In order to explore both the microeconomic and macroeconomic implications of the model, we run two different sets of simulations. For the individual results, we compute relevant statistics for each individual time series (i.e. consumption growth, volatility of consumption, etc.), reporting the average of these statistics across several consumers. For the aggregate results, first, we calculate a time series of aggregate consumption and aggregate income as the average of individual consumption and income across consumers, and then compute the relevant statistics (i.e. we aggregate explicitly).

4 Implication of Changing Down Payments

4.1 Individual Consumption

Table 1 summarizes several microeconomic consumption statistics for different down payment regimes. We report average consumption growth, as well as excess sensitivity and excess smoothness coefficients for both nondurable and durable consumption. The excess smoothness coefficient is calculated as the ratio of the standard deviation of consumption growth to that of income growth. It is therefore a measure of the relative volatility of consumption.\(^{25}\) The excess sensitivity figure is the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. It is one of the possible measures on how consumption growth reacts to predictable income changes. These statistics are computed from an individual time series of 200 periods; the table presents averages over 20,000 consumers.\(^{26}\) We also report the nondurable to durable consumption ratio, average normalized wealth (defined as the sum of financial and physical assets), percentage of voluntary equity over total wealth, and the proportion of time individuals are constrained. Panel A focuses in the no-adjustment cost case and Panel B on the adjustment cost case.

\(^{25}\) In our simulations, the volatility of income is kept constant so the excess smoothness coefficient can be used to compare absolute volatilities of consumption growth rates for different down payment regimes.

\(^{26}\) We simulate 250 periods but we ignore the first 50 to insulate the results from the influence of initial conditions. Normalized cash-on-hand converges to a stationary distribution quickly, about 12 to 15 periods starting from zero assets.
Several patterns are worth stressing.

| Table 1 |

**No Adjustment Costs**

We start with the no-adjustment cost case. First, note that with $\theta$ equal to the user cost of the durable—roughly 10 percent for our calibration—the constraint does not affect the intratemporal allocation between the durable and the nondurable. The agent spends fixed proportions of his cash-on-hand on both goods and consequently all reported statistics are identical for both goods. Second, observe that as the down payment increases, the average nondurable to durable ratio goes up since buying the durable good is more costly in terms of current liquidity. Also, the agent carries more total wealth but less voluntary equity since the durable is liquid and can be sold to free required equity without cost when necessary. In fact, for down payments higher than 30 percent, consumers hold no voluntary equity.

With respect to nondurable consumption, we observe that both consumption growth and its volatility decreases monotonically with increasing down payments. Moreover, agents can smooth nondurable consumption considerably for all possible down payments (the smoothness coefficient is well below 60 percent for down payments of 20 percent or higher). Durable consumption growth is also smoother than income. However, durable consumption growth and its volatility are non-monotonic in $\theta$, increasing with $\theta$ first and then decreasing.

There are two channels which explain the decrease in the volatility of nondurable consumption growth with increasing down payments. The first channel operates through the increase in total wealth: more wealth means agents have more resources available to smooth transitory income shocks. The second channel relates to the fact that when down payments are high, agents spread out the accumulation or liquidation of required wealth holdings in response to income shocks. For example, if there is an above-average permanent income shock, the agent’s new equilibrium level of durable consumption increases. This higher level of durable consumption, however, requires more wealth holdings in the form of the required down payment. Instead of increasing required wealth immediately to its equilibrium level, the agent accumulates the new down payment requirement over time so that nondurable consumption does not suffer temporarily. As $\theta$ increases, the burden imposed by the down payment increases resulting in more smoothing of the required down payment.
To be precise, if agents were only subject to permanent shocks, the degree of smoothing would be controlled by whether the down payment requirement is greater than, less than, or equal to the user cost of the durable, \((R - \psi)/R\). The user cost can be thought of as the durable’s long-run price in terms of the agent’s intertemporal budget constraint. When the constraint binds, the short-run cost of the durable is effectively \(\theta\), the cost of a unit of durable consumption in terms of foregone nondurable consumption. When the prices are equal and the agent is constrained, the wealth requirement imposes a cost (in terms of foregone nondurable consumption) that is the same as the user cost. This sends a signal to the agent to adjust fully to the new equilibrium level of durable consumption. When the short-run cost of the durable is greater than its long-run cost (when \(\theta > (R - \psi)/R\)), the agent gradually adjusts to the new equilibrium level of durable consumption over several periods. A full adjustment would impose too great a sacrifice of nondurable consumption relative to the unconstrained situation. When the short-run cost is lower than the long-run cost, the agent faces favorable credit conditions and consumption growth overshoots income growth. Figure 3 illustrates this point. It depicts consumption and income growth for an agent who receives average income shocks every period except for period 0, when he receives an above average permanent income shock. We consider three different down payments: 5, 10 and 30 percent. For the low down payment—which is below the user cost—consumption growth overshoots income growth. For the 10 percent down payment, consumption growth reacts one-to-one to income growth. For the high down payment, smoothing takes place.

The effect of burdensome down payments on durable consumption growth is analogous to the effect on nondurable consumption growth: higher down payments result in smoother durable growth. The non-monotonicity in durable volatility comes from the durable’s role as a store of wealth. If an agent faces a transitory negative income shock, after voluntary equity runs out, required equity holdings are liquidated and used to smooth nondurable consumption. As \(\theta\) increases, the agent carries less voluntary equity resulting in more occasions when durable consumption is reduced to convert required wealth into nondurable consumption. One may expect, then, to observe less smoothing of the durable as \(\theta\) increases. However, this is not the case for the entire range of \(\theta\). Eventually, durable consumption growth becomes smoother as the down payment increases. This is because for high values of \(\theta\), the amount of forced saving is so high that the efficiency of transforming required wealth holdings into nondurable consumption increases and requires a
less dramatic reduction in durable consumption. Since the wealth requirement is proportional to the durable stock, doubling the wealth requirement implies halving the amount of durable consumption reduction necessary to yield a given amount of liquid resources. This effect allows the agent to liquidate more wealth without as great a reduction in durable consumption. As a consequence, the volatility of durable consumption starts to fall again as the down payment requirement gets higher.

Table 1 also shows that excess smoothness of consumption growth is not associated with robust excess sensitivity. The specification for individual income growth—an MA(1)—implies a negative correlation between consumption growth and lagged income growth (a high innovation now signals low income growth next period), but as down payments become more burdensome, the sluggish response of consumption to permanent income changes generates a positive correlation. For high down payments, the effects cancel out resulting in no excess sensitivity.\(^{27}\) Finally, we emphasize that even if nondurable consumption volatility increases when down payments decrease, agents are better off under the more favorable credit conditions. In fact, nondurable consumption growth increases with decreases in \(\theta\).

\textit{Adjustment Costs}

With adjustment costs, things are just slightly more complicated. Agents change the durable stock infrequently in order to minimize the transaction cost. Just as in the no-adjustment cost case, consumers save as they anticipate the down payment requirement needed to change the durable stock. They also engage in additional minimal savings to pay for the adjustment cost. This is done very close to the period of adjustment. Figure 4 depicts a simulation in which an individual receives average income shocks for a number of periods using a down payment of 30 percent. The graph shows normalized durable and nondurable consumption, and voluntary equity. In the no-adjustment cost case the agent keeps \(c\) and \(k\) constant and carries no voluntary equity. In the adjustment cost case, \(k\) follows an \((S,s)\) rule. Even small adjustments are costly, so agents let durables depreciate until the \(s\) trigger is reached and adjustment takes place. Note that agents build up voluntary equity close to the period of adjustment, and that nondurable consumption, while relatively smooth, suffers slightly during the adjustment period.

\(^{27}\)The empirical evidence with respect to excess sensitivity at the individual level is mixed. While Hall and Mishkin (1982) find a negative correlation between consumption growth and lagged income growth, Attanasio and Weber (1995) argue that after accounting for changes in household composition and labor supply there is no evidence of excess sensitivity in U.S. micro data.
Table 1 (Panel B) shows that the $C-K$ ratio is higher with adjustment costs since the durable good is less attractive. Also, while wealth increases with the down payment, just as in the no-adjustment cost case, voluntary equity does not go all the way down to zero. It is now costly to sell the durable to free required equity so agents try to avoid this situation by keeping a small liquid buffer-stock. Just as in the no-adjustment cost case, nondurable consumption growth volatility decreases with increasing down payments, while the volatility of durable consumption growth is non-monotonic for the same reasons. However, durable consumption growth is now more volatile than income growth. This is due to the optimal $(S,s)$ adjustment rule for the durable. In the inaction region, the durable growth rate is equal to minus the depreciation rate, while during the year of adjustment durable consumption growth is quite substantial (see Figure 5). Nondurable consumption growth is still smoother than income growth, although smoothing is slightly less effective than in the no-adjustment cost case because the nondurable suffers some in the period of adjustment.

4.2 Aggregate Consumption

The main question in this paper is whether our model can account for the macroeconomic stylized facts of excess sensitivity and excess smoothness of nondurable consumption. We explore the aggregate implications of the model next. First, we present the relevant statistics for U.S. aggregate data in Table 2, and then compare them to the numbers generated by our model reported in Table 3.

**The Aggregate Excesses in the Data**

We obtain annual U.S. aggregate data from the BEA for the period 1959–2001. All flow series come from NIPA and all stock series from FADC. We use the following variables: disposable labor income, consumption expenditure on nondurables and services (with and without housing services and expenditure on clothing and shoes), and the stock of non-residential durables and residences. Disposable labor income is calculated from several NIPA components. Specifically, disposable labor income is the sum of wages and salaries plus other labor income, minus personal contributions for social insurance and taxes. Taxes are defined as the fraction of wage and salary income in total income, times personal tax and non tax payments.
type price deflator. Income is deflated by the personal consumption expenditure deflator. The excess smoothness coefficient is the ratio of consumption growth to income growth, and the excess sensitivity coefficient comes from the OLS regression of consumption growth on lagged income growth.

[Table 2]

Table 2 shows the excess coefficients for each consumption series.\(^{29}\) Note that all consumption variables exhibit smoothness relative to income except for the non-residential durable stock. Moreover, they all exhibit excess sensitivity. In the data, \(K\) is smoother than \(C\) and the excess sensitivity coefficient is higher for \(K\) than for \(C\). The aggregates that best match our model are nondurable expenditure plus services minus housing services for \(C\), and the residential stock plus the non-residential durable stock for \(K\).\(^{30}\) For these aggregates, the excess sensitivity coefficients are 0.16 for the nondurable and 0.26 for the durable. The excess smoothness ratios are 0.67 for the nondurable and 0.42 for the durable.

**The Aggregate Excesses in the Model**

Can our model reproduce these numbers? In order to determine the macroeconomic implications of the model, we explicitly aggregate over consumers who behave according to it. We generate idiosyncratic and aggregate labor income shocks from the assumed income distributions for 2,000 consumers during 200 periods. Given the calculated individual optimal consumption rules, we use the generated shocks to simulate nondurable and durable consumption for each consumer. Aggregate consumption and aggregate income in a given period are calculated as averages of consumption and income across individuals.\(^{31}\) Then, the relevant statistics (i.e., the excess smoothness ratio, the excess sensitivity coefficient, etc.) are calculated using the aggregate time series. The procedure is repeated for 100 independent simulations. Table 3 presents averages across the 100 simulations of the relevant statistics. Panel A focuses on the no-adjustment cost case while panel B focuses on the adjustment cost case.

[Table 3]

\(^{29}\)We acknowledge that referring to the excess sensitivity and excess smoothness of the durable is an abuse of terminology, as these terms refer to the empirical evidence for the nondurable. We report analogous coefficients for the durable in order to provide an initial guideline of the fit of our model to the durable as well.

\(^{30}\)In our specification, services from housing are derived from \(K\), not from \(C\).

\(^{31}\)We simulate 250 periods but the first 50 periods are discarded to insulate results from initial conditions. Using more 2,000 than consumers does not change the results.
We start with the no-adjustment cost case. Note that with aggregation, idiosyncratic shocks cancel out and what remains is the influence from the common shock, which is permanent. Not surprisingly then, when the down payment parameter is equal to the user cost, the model does not deliver either excess smoothness or excess sensitivity and the results are similar to those of the standard buffer-stock model. Ludvigson and Michaelides (2001), table 2, report an excess sensitivity coefficient of 0.001 and an excess smoothness value of 0.99 for our particular calibration, identical to ours. Moreover, when the down payment is lower than the user cost the model generates excess volatility. For down payments higher than the user cost, we obtain robust excess smoothness and in some cases excess sensitivity. As discussed in Section 4.1, agents choose not to adjust consumption levels immediately when facing permanent income shocks if down payments are burdensome, preferring to spread out the accumulation or liquidation of required wealth holdings. The higher the down payment, the longer the adjustment process. Furthermore, excess sensitivity appears because consumption optimally responds with a lag to changes in income. Note that as in the data, \( K \) is smoother than \( C \) and its excess sensitivity coefficient is higher.

Table 3 also shows that excess smoothness increases monotonically with increases in the down payment, but that excess sensitivity is non-monotonic. The best way to explain the intuition behind this finding is through the following controlled experiment. Let us simulate a representative individual who receives the aggregate income process.\textsuperscript{32} Let him receive average income shocks all periods but period 0. Figure 6 depicts the paths for the growth rates of nondurable and durable consumption for three different down payments: 10, 30 and 100 percent. Durable and nondurable consumption growth rates do not fully react to the change in income in period 0 for down payments higher than the user cost value, 10 percent. The excess sensitivity coefficient calculated in our simulations measures the reaction of current consumption changes to changes in income in the previous period only. Consider period 1 in the graph. Durable consumption growth is higher for \( \theta = 0.3 \) than for \( \theta = 1 \), so we obtain a lower excess sensitivity coefficient for the higher down payment in spite of consumption growth being smoother because of the longer adjustment period.\textsuperscript{33}

\[ \text{[Figure 6]} \]

\textsuperscript{32}With no adjustment costs, the results from our explicit aggregation and from the exercise that considers a representative agent who receives the aggregate income process are very similar. We do not tabulate the experiment here for brevity.

\textsuperscript{33}This suggests that when measuring excess sensitivity this way, one should include more lags of income growth in the specification.
Burdensome down payments can then qualitatively explain the existence of excess sensitivity and excess smoothness of nondurable consumption. Quantitatively, for $\theta = 0.3$ the smoothness ratio is 0.8; the sensitivity coefficient 0.11. While these numbers are still far from their empirical counterparts, they outperform the role of incomplete information—Ludvigson and Michaelides (2001), table 3, report an excess sensitivity coefficient of 0.104 and an excess smoothness ratio of 0.92.

With adjustment costs, the $(S,s)$ rule used by agents to adjust durable stocks implies that, unlike the no-adjustment cost case, consumption of an individual who receives aggregate income shocks may not exhibit the same properties as aggregate consumption. This is because agents adjust their durable stocks at different times (as they reach their specific triggers) when reacting to a common permanent shock. Thus, income shocks have a longer lasting effect on aggregate consumption. As a result, nondurable consumption is smoother in the adjustment cost case. In fact, for a down payment of 30 percent, the model can reproduce the actual excess sensitivity and smoothness observed in the data for the nondurable. The excess sensitivity coefficient in this case is 0.16, and the excess smoothness coefficient 0.67.

With adjustment costs, the durable also exhibits excess sensitivity and is smoother than income for down payments higher than the user cost, which was not the case at the individual level. At the aggregate level, durable consumption growth is smoother than income growth because different agents respond to the common permanent shocks at different times. In our baseline calibration, the durable is more volatile than the nondurable, unlike what we observe in the data. However, this result is not general. For example, when the transaction cost is 10 percent (see Table 4; $\phi = 0.1$), aggregate durable consumption growth is less volatile than nondurable consumption growth, in line with the data.34

Table 4 presents some further robustness analysis. We report excess smoothness and excess sensitivity coefficients for two down payments: 10 and 30 percent. The first two rows reproduce, for convenience, the results for the no-adjustment cost case and the adjustment cost case respectively. The third row considers a higher adjustment cost (10 percent), and the fourth row a lower depreciation rate for the durable, which implies a lower user cost. The last two rows explore cases with slightly different adjustment cost specifications. In row (5), the transaction cost is divided between buyers and sellers (i.e. $\zeta(K_t, K_{t-1}) = \phi_1 d\psi K_{t-1} + \phi_2 dK_t$, $\phi_1 = \phi_2 = 0.025$). In row (6), the adjustment cost is paid by the

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34This is because the higher the transaction cost, the less frequently consumers adjust their durable holdings (and the fewer consumers reach their triggers at the same time), resulting in a smaller reaction of durable consumption growth to current income growth.
seller, as in our original specification, but transaction costs are higher when investment is negative than when investment is positive (7.5 percent versus 2.5 percent). This is to allow for the possibility that some increases in the durable stock may be due to the purchase of extra units as oppose to the replacement of old ones.

In all cases, some excess smoothness is observed even for the lower down payment rate of 10 percent. For the 30 percent down payment, excess smoothness of the nondurable is similar to that observed in the data. Excess sensitivity is also the norm. The durable exhibits excess smoothness and excess sensitivity as well. Furthermore, the durable is smoother than the nondurable good as in the data. However, to match the empirical figures for durables, higher down payments are necessary.

[Table 4]

In summary, our model provides a plausible explanation for the results of excess sensitivity and excess smoothness of nondurable consumption in aggregate data. Consumption optimally responds with a lag to changes in income as consumers spread out the burden of down payments over several periods. With non-convex adjustment costs, we find an even more prolonged adjustment in the aggregate resulting in higher excess sensitivity and excess smoothness for nondurable consumption growth. In order to quantitatively match the excesses, we need a high enough down payment relative to the user cost. We do not argue that this is the only possible explanation for these so-called consumption excesses but one that should not be overlooked.

5 Conclusions

This paper studies a buffer-stock model of saving where agents consume both durable and nondurable goods, and face a minimum net worth constraint. This constraint captures the idea that durables serve as collateral. Furthermore, we consider variations of the model with and without adjustment costs in the durable market. We show that the constraint can alter the allocation of resources between the durable and the nondurable and has implications for the volatility of the two goods. Indeed, only when the down payment requirement exactly equals the user cost of the durable and there are no transaction costs should we study durables and nondurables separately.

We find that for an individual, nondurable consumption growth unambiguously becomes more volatile relative to income when the required down payment for purchases of durables in an economy is lowered. Moreover, this result is preserved by aggregation. For
the durable, the results are slightly more complex, and while some non-monotonicities exist at the individual level with respect to volatility, aggregate durable consumption growth is also more volatile for lower down payments. For an individual, this result is explained in part by the fact that higher down payment requirements translate into higher wealth holdings to deal with negative transitory income shocks—since down payments act as a form of forced saving—and in part by the fact that consumers choose to gradually adjust their consumption when facing permanent income shocks in order to spread out the burden of the down payment. At the aggregate level, it is only this last effect that survives. Moreover, this gradualism or sluggish response of consumption to permanent income shocks generates robust excess smoothness and excess sensitivity in an explicit aggregation of the model for plausible parameter values. The model with adjustment costs can match the empirical evidence for nondurable consumption.

Another interesting implication of the model is that decreases in down payments and transaction costs reduce average wealth. For example, reducing the down payment from 30 to 20 percent decreases average wealth holdings by over 30 percent (see Table 3). For a down payment of 30 percent, eliminating the adjustment cost reduces average wealth holdings by 10 percent. Differences in the durables market may play an important role in explaining differences in saving rates across countries or even the decrease in the saving rate in the U.S. in last few decades. This question, however, is better addressed in a general equilibrium setting.

We stress that lower down payment requirements imply that households voluntarily lower wealth holdings (because of their impatience) and voluntarily accept the cost of greater volatility. In this model, *ceteris paribus*, consumers should be better-off in an economy with lower down payments.

Several factors not included in the model may further refine the results of this paper and deserve attention for future research. First, some consumers may simply give up saving for down payments if the cost is too prohibitive. An interesting extension to the current model would be to allow for this phenomenon which would require the explicit presence of a rental market. Second, the model would benefit from the inclusion of stochastic durable prices and its extension to a general equilibrium setting.\(^{35}\)

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\(^{35}\text{In a related paper, Díaz and Luengo-Prado (2002) find that decreasing down payments or transaction costs lead to slightly higher real interest rates in general equilibrium. The increase in the interest rate (which increases the user cost) is not big enough to overturn the results in this paper.}\)
Appendix A. Numerical Procedures

A.1 Euler Equation Iteration

Derivation of key equations (10) and (11)

Using our particular utility specification, we can write equations (5) and (6), the Euler-Lagrange necessary conditions, as follows:

\[-C_t^{-\rho} + \beta R E_t \left\{ [C_{t+1}(X_{t+1})]^{-\rho} \right\} + \lambda_t = 0, \tag{5'}\]

\[\varphi K_t^{-\rho} - \beta (R - \psi) E_t \left\{ [C_{t+1}(X_{t+1})]^{-\rho} \right\} - \theta \lambda_t = 0. \tag{6'}\]

As in Carroll (1997), we rewrite the equations in ratio terms by taking advantage of the homogeneity of degree \(\rho\) of marginal utility and dividing all variables by permanent income:

\[-c_t^{-\rho} + \beta R E_t \left\{ (G_{t+1}N_{t+1})^{-\rho} [c_{t+1}(x_{t+1})]^{-\rho} \right\} + \lambda_t P_t^o = 0, \tag{5''}\]

\[\varphi k_t^{-\rho} - \beta (R - \psi) E_t \left\{ (G_{t+1}N_{t+1})^{-\rho} [c_{t+1}(x_{t+1})]^{-\rho} \right\} - \theta \lambda_t P_t^o = 0, \tag{6''}\]

where

\[x_{t+1} \equiv \frac{X_{t+1}}{P_{t+1}} = (G_{t+1}N_{t+1})^{-1} [R(x_t - c_t) + (\psi - R)k_t] + V_{t+1},\]

\[c_t \equiv C_t/P_t\] and \(k_t \equiv K_t/P_t\).

When the agent is not liquidity constrained, \(\lambda_t = 0\). Moreover, as we know from the intratemporal condition, \(c_t/k_t = \Omega\). Using these facts and the definition of \(x_{t+1}\) above, equation (5'') can be written as:

\[\beta R E_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left( (G_{t+1}N_{t+1})^{-1} \left( R(x_t - c_t) + \left( \frac{\psi - R}{\Omega} \right) c_t \right) + T_{t+1} \right) \right) \right\}^{-\rho} - c_t^{-\rho} = 0. \tag{10}\]

When the agent is constrained, \(x_t = c_t + \theta k_t\). Also, we can solve for \(\lambda_t P_t^o\) in equation (5''). Substituting into equation (6''), we can write:
\[
\beta[\psi - R(1 - \theta)] E_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1}[\psi - R(1 - \theta)]^{\frac{x_t - c_t}{\theta}} + T_{t+1} \right] \right)^{-\rho} \right\} \\
- \theta c_t^{-\rho} + \varphi \left( \frac{x_t - c_t}{\theta} \right)^{-\rho} = 0. \tag{11}
\]

**About the Technique**

With the two equations above, we can find the optimal rule for normalized nondurable consumption as a function of the unique state variable, normalized cash-on-hand, \( x \). We denote the optimal rule as \( c(x) \). Euler equation iteration requires assuming a finite horizon, \( T \), and recursively solving backwards from the last period of life. To apply the method successfully, we need to (i) evaluate the expectation, (ii) select an appropriate terminal condition and (iii) find a criterion to check if the agent is constrained.

In order to evaluate the expectation, we avoid numerical integration by replacing the continuous \( G_t, N_t \) and \( T_t \) processes by 5-point discrete approximations as suggested by Tauchen (1986). With regards to the terminal condition, we assume that, as in Deaton (1992), the value of total assets is zero at time \( T \), \( a_T + k_T = 0 \). Then \( c_T(x) = x \) (the agent spends all his cash-on-hand on the nondurable).

In period \( T - 1 \), for a given value of \( x \), we can numerically compute the value \( c_{T-1} \) that satisfies the appropriate equation: the first one if the agent is not constrained and the second one if he is. We do so for a grid of values of \( x \) and numerically approximate the optimal consumption rule \( c_{T-1}(x) \) through interpolation between the points of the \( x \) grid (we use cubic spline interpolation). Once we have \( c_{T-1}(x) \), the same grid of \( x \) values is used to compute \( c_{T-2}(x) \). With \( c_{T-2}(x) \), \( c_{T-3}(x) \) is computed, and so on.

Note that there is a shortcut to verify if the agent is constrained for a given value of \( x \). At each time iteration, find \( x_t^*(\theta) \), the exact value of cash-on-hand for which the liquidity constraint just binds. This can be done by noticing that at this point, \( x_t^*(\theta) = c_t(x_t^*(\theta)) + \theta k_t(x_t^*(\theta)) \) and \( c_t(x_t^*(\theta)) = \Omega k_t(x_t^*(\theta)) \). This implies that \( c_t(x_t^*(\theta)) = \Omega(\Omega + \theta)^{-1} x_t^*(\theta) \). Then we can just solve equation (10) for \( x_t^*(\theta) \). For all \( x \leq x_t^*(\theta) \) the agent is constrained, and vice versa.

Once we have the optimal policy function for the nondurable, \( c(x) \), the optimal policy function for the durable, \( k(x) \), can be calculated by using the intratemporal relationship
between the two goods.

\[ k(x) = \begin{cases} 
\theta^{-1}[x - c(x)], & x \leq x^*(\theta), \\
\Omega^{-1}c(x), & x > x^*(\theta).
\end{cases} \]

**Convergence Conditions**

Two sufficient conditions for the individual Euler equations (5) and (6) to define a contraction mapping for \( \{c(x), k(x)\} \) are the conditions in Theorem 1 of Deaton and Laroque (1992). In our case:

\[ \beta RE_t[(G_{t+1}N_{t+1})^{-\rho}] < 1, \]  
\[ \beta (R - \psi)E_t[(G_{t+1}N_{t+1})^{-\rho}] < 1. \]  

Equation (17) is the “impatience” condition derived by Deaton (1991) with \( \mu_N = 0 \). This condition ensures that borrowing is part of the unconstrained plan. For equation (18) to be satisfied, \( R > \psi \). Moreover, as long as \( 0 < R - \psi < 1 \), condition (17) is stricter than condition (18). Briefly, for \( 0 < R - \psi < 1 \), the standard impatience condition common to buffer-stock models guarantees convergence. For \( R - \psi > 1 \), convergence is guaranteed by condition (18). For \( R < \psi \), convergence is not guaranteed.

**A.2 Finite State Approximation**

The technique consists of specifying a finite-state problem that approximates the continuous one we want to solve. We replace the continuous state variables, \( k \) and \( q \), with the finite sets, \( K = \{k_1, \ldots, k_{N_k}\} \) and \( Q = \{q_1, \ldots, q_{N_q}\} \). Note that the problem has been conveniently formulated in such a way that the control variables are the next period’s states. The liquidity constraint is implemented by setting \( q_1 = 0 \) and \( q_i > 0, \forall q_i \in Q, i > 1 \). To deal with adjustment cost, we set:

\[ \frac{1}{\rho} [\ln(\beta) + \ln(R)] + \frac{\rho}{2} (\sigma_G^2 + \sigma_N^2) < \mu_G, \]
\[ \frac{1}{\rho} [\ln(\beta) + \ln(R - \psi)] + \frac{\rho}{2} (\sigma_G^2 + \sigma_N^2) < \mu_G. \]
\[ d = \begin{cases} 0, & |k_t - (G_t N_t)^{-1} \psi k_{t-1}| \leq \kappa, \\ 1, & |k_t - (G_t N_t)^{-1} \psi k_{t-1}| > \kappa, \end{cases} \]

where \( \kappa = (k_n - k_l)/(N_k - 1) \). The precision of our solution increases as \( \kappa \) falls. This “work around” solution may have some economic significance. It may be possible for the agent to make small changes to his durable stock, such as repairs, which do not require significant adjustment costs. If this is the case, the numerical formulation described here would be most appropriate.

As with the previous technique, all components of the income process are discretized. \( N_G \) points for \( G \), \( N_N \) points for \( N \), and \( N_T \) points for the transitory shock \( T \). We then use value function iteration, which is sped up with an acceleration technique, modified policy function iteration with \( S \) states.\(^{36}\) Briefly,

1. Choose an initial guess \( V^0 \). Let \( V^\ell = V^0 \).

2. Calculate \( U^{\ell+1} = UV^\ell \). For each \((q_i, k_j)\), the mapping \( U \) is defined as:

\[
U^{\ell+1}_{i,j,m,n,o} = \max_{q^+, k^+, q^+ \geq 0} U \left( (G_m N_n)^{-1} \left\{ Rq_i + [\psi(1 - d\phi) - R(1 - \theta)]k_j \right\} - \theta k^+ + T_o - q^+, k^+ \right) + V^\ell(q^+, k^+) \equiv UV^\ell_{i,j}.
\]

3. Let \( W^0 = V^\ell \). For each \((q_i, k_j)\) and \( s = 1, \ldots, S \), calculate:

\[
W^{s+1}(q_i, k_j) = \frac{1}{N_G N_N N_T} \sum_{m=1}^{N_G} \sum_{n=1}^{N_N} \sum_{o=1}^{N_T} (G_m N_n)^{-1} \left\{ Rq_i + [\psi(1 - d\phi) - R(1 - \theta)]k_j \right\} - \theta U^{\ell+1}_k + V^\ell - U^{\ell+1}_{e+1} + W^{s}(U^{\ell+1}_e, U^{\ell+1}_k) + W^{s}(U^{\ell+1}_q, U^{\ell+1}_k) \right) \right) .
\]

Set \( V^{\ell+1} = W^S \).

4. Iterate until convergence.

Note that the selection of appropriate bounds for the sets \( K \) and \( Q \) is key for the successful application of the technique. See Farr and Luengo-Prado (1999) for more information about this method.

\(^{36}\)Every time a new policy function is computed, we calculate the value function that would result from using this policy function \( S \) times. With the newly obtained value function, we compute the new optimal policy function and so on. See Judd (1997) for a general description of these procedures.
For the construction of Table 3, we set $N_G = N_N = N_V = 5$; $N_k = 150$, $k_1 = 0.01$ and $k_{N_k} = 2.5$; $N_q = 70$, $q_1 = 0$ and $q_{N_q} = 0.3$ for $\theta \in [0, 0.4]$; $N_q = 90$, $q_1 = 0$ and $q_{N_q} = 0.4$ for $\theta = 0.5$; $N_q = 110$, $q_1 = 0$ and $q_{N_q} = 0.5$ for $\theta = 1$.

A.3 Comparing Techniques

We solve the problem without adjustment costs for a few parameter values with both techniques. Results are shown in Table 5. All statistics are fairly similar, which suggests that the finite state approximation technique produces reasonable results. With regards to computational time, the procedures rank very differently. Convergence of a policy function using Euler equation iteration (EEI) takes between 25 and 40 iterations and a total time of 0.45 minutes. A typical simulation takes 0.8 minutes. Convergence of policy functions using the finite state approximation (FSA) takes between 10 and 12 iterations and 35 minutes. A typical simulation takes 0.7 minutes.\(^{37}\) EEI is a faster procedure but it cannot be used when introducing adjustment costs. Moreover, it must be tailored to deal with other specifications of utility functions and its implementation may not be feasible in all cases. FSA is a more robust method. It can easily accommodate adjustment costs and different utility specifications.

[Table 5]

Appendix B. Proofs of Propositions

Proof of Proposition 1

We know from the intratemporal first order condition of the problem that the ratio of nondurable to durable consumption equals $\Omega$, regardless of the value of $\theta$. Moreover $x_t = c_t + \theta k_t + q_t$. Both conditions imply the results.

Proof of Proposition 2

(A) If $\theta = 0$, $x_t = c_t + q_t$. When the agent is constrained, $q_t = 0$ so $c(x) = x$. $k(x)$ does not depend on $x$ in this case since it does not impose any cost on current liquidity. Note that $k$ provides utility today but decreases cash-on-hand tomorrow ($\psi < R$). Therefore, there is an optimal level of $k$ while $x$ is below $x^*(\theta)$. Given that $k(x) = \Omega^{-1} (x - q)$ for $x \geq x^*(\theta)$, $k(x) = \Omega^{-1} x^*(\theta)$ for $x < x^*(\theta)$.

\(^{37}\) All programs are in C++. Calculations were performed using an Intel Xeon 2.8 GHz processor.
(B) Since the agent is liquidity constrained, \( x = c(x) + \theta k(x) \). Then, \( 1 = c'(x) + \theta k'(x) \), \( 0 = c''(x) + \theta k''(x) \), with \( c'(x) > 0 \) and \( k'(x) > 0 \), and \( c(0) = k(0) = 0 \). From the intratemporal first order condition we know that:

\[
\frac{c(x^*(\theta))}{k(x^*(\theta))} = \Omega \quad \text{and} \quad \frac{c(x)}{k(x)} < \Omega, \quad \forall x < x^*(\theta).
\]

The closer \( x \) to \( x^*(\theta) \), the lower the shadow price of the constraint. Therefore the \( c-k \) ratio increases with \( x \). The only way this can happen is with \( c''(x) > 0 \) and \( k''(x) < 0 \). Hence \( c(x) \) must be convex and \( k(x) \) concave while the agent is constrained.

(C) We observe from the intratemporal first order condition that in this case, the agent is able to keep the \( c-k \) ratio constant and equal to \( \Omega \). Moreover \( c(x) + \theta k(x) = x \). Both conditions imply the result.

(D) The proof is similar to (B). In this case, the \( c-k \) ratio is higher than \( \Omega \) when the agent is constrained and decreases towards \( \Omega \) as \( x \) approaches \( x^*(\theta) \). The only way this can happen is with a concave \( c(x) \) and a convex \( k(x) \).
References


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Table 1: Microeconomic Results from Individual Time Series

<table>
<thead>
<tr>
<th>θ</th>
<th>0.0</th>
<th>0.05</th>
<th>~ 0.1◊</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>1.0</th>
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<tbody>
<tr>
<td>Avg. $g_C$ (%)</td>
<td>2.49</td>
<td>2.45</td>
<td>2.38</td>
<td>2.27</td>
<td>2.21</td>
<td>2.17</td>
<td>2.15</td>
<td>2.11</td>
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<td>$g_C$ smoothness</td>
<td>0.88</td>
<td>0.83</td>
<td>0.76</td>
<td>0.63</td>
<td>0.54</td>
<td>0.47</td>
<td>0.43</td>
<td>0.36</td>
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<td></td>
<td>(0.039)</td>
<td>(0.033)</td>
<td>(0.025)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$g_C$ sensitivity</td>
<td>-0.171*</td>
<td>-0.167*</td>
<td>-0.141*</td>
<td>-0.085</td>
<td>-0.038</td>
<td>-0.008</td>
<td>0.004</td>
<td>0.014</td>
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<td></td>
<td>(0.061)</td>
<td>(0.058)</td>
<td>(0.053)</td>
<td>(0.045)</td>
<td>(0.038)</td>
<td>(0.034)</td>
<td>(0.031)</td>
<td>(0.026)</td>
</tr>
</tbody>
</table>

| Avg. $g_K$ (%) | 2.22 | 2.29 | 2.38  | 2.45 | 2.39 | 2.29 | 2.23 | 2.11 |
| $g_K$ smoothness | 0.56 | 0.65 | 0.76  | 0.84 | 0.78 | 0.66 | 0.57 | 0.35 |
|                | (0.024) | (0.022) | (0.025) | (0.021) | (0.017) | (0.016) | (0.015) | (0.015) |
| $g_K$ sensitivity | -0.053 | -0.106* | -0.141* | -0.136* | -0.077 | -0.018 | 0.009 | 0.034 |
|                | (0.040) | (0.046) | (0.053) | (0.059) | (0.055) | (0.047) | (0.041) | (0.025) |

| C-K ratio | 0.35 | 0.35 | 0.36  | 0.37 | 0.38 | 0.39 | 0.40 | 0.45 |
| Avg. wealth | 0.03 | 0.13 | 0.24  | 0.43 | 0.63 | 0.82 | 1.00 | 1.82 |
| Avg. $q$ (%) | 100.0 | 17.4 | 7.2   | 1.7  | 0.2  | 0.0  | 0.0  | 0.0  |
| Constrained (%) | 40.9 | 43.1 | 52.2  | 72.7 | 91.1 | 98.5 | 99.8 | 100.0 |

Panel A. No Adjustment Costs

| Avg. $g_C$ (%) | 2.47 | 2.45 | 2.43  | 2.39 | 2.34 | 2.32 | 2.29 | 2.25 |
| $g_C$ smoothness | 0.86 | 0.85 | 0.83  | 0.78 | 0.73 | 0.71 | 0.67 | 0.60 |
|                | (0.042) | (0.042) | (0.043) | (0.044) | (0.044) | (0.045) | (0.043) | (0.038) |
| $g_C$ sensitivity | -0.161* | -0.153* | -0.135* | -0.080 | -0.030 | 0.015 | 0.038 | 0.068 |
|                | (0.060) | (0.046) | (0.053) | (0.059) | (0.055) | (0.052) | (0.050) | (0.048) |

| Avg. $g_K$ (%) | 4.63 | 4.70 | 4.76  | 4.71 | 4.52 | 4.28 | 4.07 | 3.35 |
| $g_K$ smoothness | 2.37 | 2.41 | 2.45  | 2.42 | 2.31 | 2.16 | 2.04 | 1.57 |
|                | (0.130) | (0.135) | (0.139) | (0.136) | (0.127) | (0.117) | (0.110) | (0.083) |
| $g_K$ sensitivity | -0.042 | 0.004 | 0.060  | 0.117 | 0.118 | 0.108 | 0.084 | 0.098 |
|                | (0.169) | (0.172) | (0.174) | (0.172) | (0.164) | (0.154) | (0.145) | (0.112) |

| C-K ratio | 0.39 | 0.39 | 0.39  | 0.40 | 0.41 | 0.43 | 0.44 | 0.49 |
| Avg. wealth | 0.03 | 0.13 | 0.24  | 0.45 | 0.66 | 0.86 | 1.05 | 1.86 |
| Avg. $q$ (%) | 100.0 | 20.2 | 12.9  | 10.9 | 10.9 | 10.9 | 10.9 | 9.7 |
| Constrained (%) | 43.1 | 44.2 | 43.5  | 40.9 | 38.0 | 34.8 | 31.8 | 30.7 |

Panel B. Adjustment Costs

Notes: $g_C$ and $g_K$ are the growth rate of nondurable consumption and durable consumption respectively. The rows labelled “smoothness” report the ratio of the standard deviation of (nondurable or durable) consumption growth to the standard deviation of income growth. The rows labelled “sensitivity” report the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. The row labelled “avg. wealth” presents normalized wealth, the sum of financial wealth plus durable wealth divided by permanent income ($a + k$). $q$ is normalized voluntary equity, the wealth held in excess of the required down payment. The row labelled “avg. $q$ (%)” is the percentage of voluntary equity on total wealth. In all cases, $R = 1.02$, $\psi = 0.915$, $\varphi = 0.795$, $\beta = 1/1.05$ and $\rho = 2$. $\phi = 0.05$ in the adjustment cost case. The parameters for the different income shocks are: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_T = 0$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. These imply a growth rate for labor income of 2.63 percent and a standard deviation of 11.07 percent. The statistics reported are calculated from an individual time series of 200 periods; the table shows averages over 20,000 individuals. In parentheses, we report the standard deviation of the smoothness ratio across the 20,000 individuals, as well as the average standard error of the regression coefficient of the excess sensitivity parameter.

◊ down payment equal to user cost.

* significant at 5%.
Table 2: Stylized Facts of U.S. Data. 1959–2001

<table>
<thead>
<tr>
<th></th>
<th>Excess Smoothness</th>
<th>Excess Sensitivity</th>
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<tr>
<td></td>
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</tr>
<tr>
<td>$C$</td>
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<td>nondurables+services</td>
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<td>0.18</td>
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<tr>
<td></td>
<td>(1.71)</td>
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</table>

| $K$                      |                   |                    |
| non-residential durable stock | 0.98             | 0.66               |
|                          | (5.78)            |                    |
| residential durable stock  | 0.36              | 0.19               |
|                          | (3.78)            |                    |
| non-residential+residential stock | 0.42            | 0.26               |
|                          | (4.93)            |                    |

Notes: *Excess Smoothness* is measured as the ratio of the standard deviation of consumption growth to the standard deviation of the income growth. *Excess Sensitivity* refers to the OLS coefficient from a regression of consumption growth on lagged income growth and a constant (t-statistic in parentheses). All flow series are from the *National Product and Income Accounts* and all stock series from *Fixed Assets and Consumer Durable Goods Accounts* published by the Bureau of Economic Analysis. All series are per capita and appropriately deflated.
### Table 3: Macroeconomic Results from Aggregate Time Series

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<th>θ</th>
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<td>2.17</td>
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<td>2.16</td>
<td>2.16</td>
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<td>2.17</td>
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<td>2.15</td>
<td>2.15</td>
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<td>1.04</td>
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<td>0.66</td>
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<td>φ</td>
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<td>0.72</td>
<td>0.96</td>
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</tr>
<tr>
<td>Avg. $q$ (%)</td>
<td>100.0</td>
<td>19.7</td>
<td>12.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>9.1</td>
</tr>
<tr>
<td>φ</td>
<td>0.9</td>
<td>0.95</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
<td>1.25</td>
<td>1.3</td>
<td>1.6</td>
</tr>
</tbody>
</table>

**Notes:** $g_C$ and $g_K$ are the growth rate of nondurable consumption and durable consumption respectively. The rows labelled “smoothness” report the ratio of the standard deviation of (nondurable or durable) consumption growth to the standard deviation of income growth. The rows labelled “sensitivity” report the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. The row labelled “avg. wealth” presents normalized wealth, the sum of financial wealth plus durable wealth divided by permanent income $(a + k)$. $q$ is normalized voluntary equity, the wealth held in excess of the required down payment. The row labelled “avg. $q$ (%)” is the percentage of voluntary equity on total wealth. In all cases, $R = 1.02$, $\psi = 0.915$, $\beta = 1/1.05$ and $\rho = 2$. $\phi = 0.05$ in the adjustment cost case. The parameters for the different income shocks are: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_T = 0$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. These imply a growth rate for income of 2.17 percent and a standard deviation of 2.5 per cent. $\varphi$, the preference parameter for durable goods, is allowed to vary with $\theta$ to keep the $C-K$ ratio constant and equal to 0.36. The statistics reported are calculated from an aggregate time series of 200 periods. Aggregate consumption and income are calculated as averages over 2,000 individuals. We report average statistics for 100 independent simulations. In parentheses, we present the standard deviation of the smoothness ratio across the 100 simulations, as well as the average standard error of the regression coefficient of the excess sensitivity parameter. * significant at 5% level.

◊ down payment equal to user cost.
Table 4: Further Results from Aggregate Time Series

<table>
<thead>
<tr>
<th></th>
<th>( g_C )</th>
<th>( g_K )</th>
<th>( g_C )</th>
<th>( g_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex. sen</td>
<td>Ex. sm.</td>
<td>Ex. sen</td>
<td>Ex. sm.</td>
<td></td>
</tr>
<tr>
<td>( \theta = 0.1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) ( \phi = 0 )</td>
<td>0.00</td>
<td>0.99</td>
<td>0.00</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>(2) ( \phi = 0.05 )</td>
<td>0.01</td>
<td>0.91</td>
<td>0.21*</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.01)</td>
<td>(0.07)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>(3) ( \phi = 0.1 )</td>
<td>0.00</td>
<td>0.92</td>
<td>0.30*</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.01)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(4) Higher ( \psi )</td>
<td>0.04</td>
<td>0.87</td>
<td>0.30*</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.01)</td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>(5) Buyer-Seller</td>
<td>0.01</td>
<td>0.88</td>
<td>0.31*</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(6) Seller &gt; Buyer</td>
<td>0.10</td>
<td>0.82</td>
<td>0.19</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.03)</td>
<td>(0.13)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

\( \theta = 0.3 \)

<table>
<thead>
<tr>
<th></th>
<th>( g_C )</th>
<th>( g_K )</th>
<th>( g_C )</th>
<th>( g_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex. sen</td>
<td>Ex. sm.</td>
<td>Ex. sen</td>
<td>Ex. sm.</td>
<td></td>
</tr>
<tr>
<td>(1) ( \phi = 0 )</td>
<td>0.11*</td>
<td>0.80</td>
<td>0.21*</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>(2) ( \phi = 0.05 )</td>
<td>0.16*</td>
<td>0.67</td>
<td>0.38*</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(3) ( \phi = 0.1 )</td>
<td>0.10*</td>
<td>0.68</td>
<td>0.43*</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>(4) Higher ( \psi )</td>
<td>0.08</td>
<td>0.71</td>
<td>0.38*</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(5) Buyer-Seller</td>
<td>0.11*</td>
<td>0.69</td>
<td>0.38*</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(6) Seller &gt; Buyer</td>
<td>0.14</td>
<td>0.69</td>
<td>0.27*</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.04)</td>
<td>(0.09)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

Notes: \( g_C \) and \( g_K \) are the growth rate of nondurable consumption and durable consumption respectively. All parameters as in Table 3, except the parameter being changed. For row (3), \( \phi = 0.1 \); for row (4), \( \psi = 0.943 \), which implies a depreciation rate of 5.7 percent; \( \phi = 0.05 \). For row (5), we use an alternative adjustment cost specification with the transaction cost divided between buyer and seller: \( \zeta(K_t, K_{t-1}) = \phi_1 d\psi K_{t-1} + \phi_2 dK_t \), with \( \phi_1 = \phi_2 = 0.025 \). In row (6), the transaction cost is higher for negative investment than for positive investment (7.5 percent versus 2.5 percent).

* significant at 5% level.
Table 5: Comparison of Techniques. Results for the No-Adjustment Cost Model.

*(Individual Level)*

<table>
<thead>
<tr>
<th></th>
<th>0.05</th>
<th>~ 0.1°</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EEI</td>
<td>FSA</td>
<td>EEI</td>
</tr>
<tr>
<td>$c$ avg.</td>
<td>0.7771</td>
<td>0.7771</td>
<td>0.7795</td>
</tr>
<tr>
<td>$c$ sd.</td>
<td>0.0423</td>
<td>0.0424</td>
<td>0.0391</td>
</tr>
<tr>
<td>$g_C$ avg.</td>
<td>0.0245</td>
<td>0.0245</td>
<td>0.0238</td>
</tr>
<tr>
<td>$g_C$ sd.</td>
<td>0.0922</td>
<td>0.0924</td>
<td>0.0843</td>
</tr>
<tr>
<td>smoothness</td>
<td>0.8334</td>
<td>0.8356</td>
<td>0.7619</td>
</tr>
<tr>
<td>sensitivity</td>
<td>-0.1667</td>
<td>-0.1670</td>
<td>-0.1412</td>
</tr>
<tr>
<td>$g_K$ avg.</td>
<td>2.1879</td>
<td>2.1879</td>
<td>2.1661</td>
</tr>
<tr>
<td>$g_K$ sd.</td>
<td>0.0781</td>
<td>0.0787</td>
<td>0.1088</td>
</tr>
<tr>
<td>smoothness</td>
<td>0.6499</td>
<td>0.6524</td>
<td>0.7619</td>
</tr>
<tr>
<td>sensitivity</td>
<td>-0.1056</td>
<td>-0.1024</td>
<td>-0.1412</td>
</tr>
<tr>
<td>$C/K$</td>
<td>0.3503</td>
<td>0.3549</td>
<td>0.3598</td>
</tr>
</tbody>
</table>

Notes: $\theta$ is the down payment parameter. EEI stands for Euler equation iteration. FSA stands for finite state approximation. $k$ is normalized durable and $c$ is normalized nondurable. $g_K$ and $g_C$ are the growth rates of durable and nondurable consumption (in levels) respectively. $C/K$ is the ratio of nondurable to durable (in levels). ♦ denotes the value of the down payment equal to the user cost, roughly 0.1. For all cases, $R=1.02$, $\psi=0.915$, $\beta=1/1.05$, $\rho=2$, $\varphi=0.795$. The income statistics are as follows: $\mu_G=0.02$, $\sigma_G=0.025$, $\mu_N=\mu_V=0$, $\sigma_N=0.05$, and $\sigma_V=0.07$. The statistics reported are calculated from an individual time series of 200 periods; the table shows averages over 20,000 individuals. In parentheses, we report the standard deviation of the smoothness ratio across individuals, as well as the average standard error of the regression coefficient of the excess sensitivity parameter.
Figure 1: Policy Functions. Nondurable plus Down Payment as a Function of Cash-on-Hand.
Figure 2: Policy Functions. Durable and Nondurable as a Function of Cash-on-Hand
Figure 3: Adjustment to Permanent Shocks
Figure 4: Consumption and Voluntary Equity Paths with Average Shocks. 30 percent Down Payment

Figure 5: Individual Consumption and Income Growth Rates with Adjustment Costs. 30 percent Down Payment
Figure 6: Explaining Smoothness and Sensitivity.