FISCAL-MONETARY POLICY COORDINATION
AND DEBT MANAGEMENT:
A TWO STAGE DYNAMIC ANALYSIS

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Abstract. This paper studies the interaction between two autonomous policymakers, the central bank and the government, in managing public debt as the result of a two-stage game. In the first stage the institutional regime is established. This determines the equilibrium solution to be applied in the second stage, in which a differential game is played between the two policymakers. It is shown that, if the policymakers can communicate before the game is played, (multiple-equilibrium) coordination problems can be solved by using the concept of correlated equilibrium. Unlike Nash equilibrium, which only allows for individualistic and independent behaviour, a correlated equilibrium allows for the players’ behaviour to be coordinated and correlated.

Keywords: monetary and fiscal policies, differential games, correlated equilibrium.

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Fiscal-Monetary Policy Coordination and Debt Management: A Two-Stage Dynamic Analysis

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1. Introduction

In the last two decades the issue of central bank independence has been extensively analyzed in the literature. In many countries monetary policy is not directly controlled by the government and a certain degree of independence is granted to the central bank. Having an independent central bank has been interpreted as a solution to the lack of credibility of the government’s anti-inflationary policies. The idea is that an independent central bank would be mainly concerned with monetary stability.1

The independence of the central bank implies that economic policy making can be analyzed as the interaction of two autonomous decision makers with (partially) conflicting objectives. Game theoretic analyses of the interactions between monetary and fiscal policies have now become common in the literature.2 In this context, institutions can be seen as the rules of the game played by the central bank and the government.3

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1 A number of empirical studies support the proposition that central bank independence and low rates of inflation are correlated (see for example Berger et al., 2001). However, it has been argued (see, e.g., Hayo and Hefeker, 2002) that this correlation does not indicate causality and that the reasons why central banks are made independent are related to legal, cultural, political, and economic factors.

2 Seminal studies are those of Sargent and Wallace (1981), Tabellini (1986), Alesina and Tabellini (1987) and Turnovsky et al. (1988). More recent contributions are, among others, Levine and Brociner (1994), Neck and Dockner (1995). Particular emphasis has been recently placed on the problem of macroeconomic policy coordination in a monetary union (see, e.g., Cooper and Kempf 2000, Beetsma et al. 2001, Buti et al. 2001, Beetsma and
The equilibrium of the game can thus be found under different institutional arrangements, i.e. under different assumptions about the timing of play, the information that each policymaker has when it is its turn to choose and the possibility of cooperation between the policymakers.

In this paper the institutional set-up is decided by the two policymakers before the game is played. Institutional design is thus interpreted as the policymakers’ acts that determine the rules of the game (cf. Ecchia and Mariotti, 1997). Accordingly, we describe institution design as the first stage of a two-stage game.4

Attention is focused on four possible institutional arrangements (i.e. policy regimes). In the first regime the two policymakers act simultaneously (Nash). In the second the monetary authority is given first-mover advantage (monetary leadership). In the third the central bank responds to the budget decisions of the fiscal authority and it is committed to meeting the financial needs of the government (fiscal leadership). The last regime considered is one in which both authorities try to lead the game (warfare). We consider the four regimes as possible outcomes of a first stage in which each policymaker has two feasible strategies: be the leader or follow the leader. If both authorities act as followers, the solution to be applied in the second stage is a Nash equilibrium in which players take their decisions simultaneously, i.e. without knowing the reaction of the opponent. If one authority acts as leader while the other acts as follower, the solution is a leadership equilibrium in which the leader, in computing its optimal policy, takes the opponent’s reaction into account. If both authorities act as leaders, the solution is the so-called “(Stackelberg) warfare.”5 Which solution is the most appropriate description of monetary-fiscal interaction is an open

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3 See North (1990) for a discussion of institutions as rules of the game.
4 Two-stage games have been recently used in order to establish the institutional arrangements (first stage) separately from the “main” game that in such arrangements is played (second stage). An example is provided by non-cooperative endogenous coalition theory: in the first stage coalitions are formed, whereas in the second stage coalitions play the game (Ray and Vohra 1999). van Aarle et al. (2002) provide an economic application of this approach to the coordination of fiscal and monetary policies in the EMU.
5 On the interpretation of the warfare equilibrium there is an extensive debate, its description is outside the scope of this paper (see, among the others, d’Aspremont and Gérard-Varet 1980; Dowrick 1986; Hamilton and Slutsky 1990).
question. A leadership game seems a reasonable assumption because of differences in the decision and implementation processes of the two policies. Beetsma and Bovenberg (1998) argue that fiscal authorities have a first-mover advantage because fiscal policy cannot be adjusted as quickly as monetary policy. In other words, the rigidity in the decision process of fiscal policy is similar to a commitment technology. Although fiscal and monetary policies certainly have different timings, the assumption of fiscal leadership remains a pure conjecture. If one interprets leadership as deriving from the ability to pre-commit, then monetary leadership is an equally plausible scenario.6

We argue that the appropriate solution concept for the first stage (when institutions, i.e. the rules for the second stage, are established) is not Nash equilibrium, which only allows for individualistic and independent behaviour, but rather correlated equilibrium which allows for the players’ behaviour to be coordinated and correlated.7

The easiest way to think of a correlated equilibrium is to imagine that there is an external referee who suggests to each player which action he should take. If the players follow the suggestion, then the result is a correlated strategy. Aumann (1987) justifies the notion of correlated equilibrium as a result of Bayesian rationality. However, correlated equilibrium has also been justified as the equilibrium outcome in non-cooperative games with pre-play communication (Forges 1990, Lehrer 1996, Ben Porath 1998, Moreno and Wooders 1998) and as the limit distribution in learning models (Foster and Vohra 1997, and Hart and Mas-Colell 2000 and 2001). In the context of monetary-fiscal interactions the correlated device can be thought of as being a formal or informal agreement that tells the policymakers in which circumstances each of them, if any, is going to impose discipline on the other. One way to interpret this agreement is as the result of pre-play communication. Another (dynamic) interpretation, along the lines of the learning literature, would be to say that different institutions emerge as time passes. Alternatively, we can think of the correlated device as the observation of the shocks which hit the economy.

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6 See discussion in Debrun (2000) and Dixit and Lambertini (2000).
7 A formal definition of correlated equilibrium is given in section 5. The references for correlated equilibrium are Aumann (1974 and 1987).
The outcome of the first stage determines the institutional arrangement, i.e. the equilibrium solution to be applied in the second stage. In this stage a differential game is played between the central bank and the government. At this stage monetary and fiscal policies are set by different authorities; however, their decisions are subject to the government’s (dynamic) budget constraint. If public debt enters both policymakers’ loss function then the central bank would like to see the burden of reducing the debt borne mainly by the government and at the same time the government would like the central bank to accommodate its financial needs.

The literature has mainly focused on the effects of monetary-fiscal interactions on short-run stabilisation of output and inflation. In these models, explicit consideration of the government’s budget constraint is usually not introduced because it generates structural dynamics which makes the analysis much more complicated.\(^8\)

In this paper, attention is focused on monetary-fiscal interactions stemming from their effects on public debt. We follow Tabellini (1986) and apply a two-step procedure which, by separating the effects of monetary-fiscal interactions on public debt from those on output and inflation, greatly simplifies the dynamics of the model.\(^9\) The first step of the procedure consists of computing the optimal policies in the absence of public debt. These unconstrained strategies have to be modified to take into account the effects on public debt dynamics. The adoption of this procedure allows us to write down a model in which the policymakers’ loss functions are directly defined on the policy instruments and on the state variable (public debt).

Since our concern is not with the transitory effects of fiscal and monetary impulses, but rather with the permanent effects of money growth and deficit levels, we take a long run perspective and compute the steady state solution of the differential game. Transition properties of these kind of model have been already studied in details (see, e.g., Tabellini 1986; and van Aarle et al., 1997).

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\(^8\) In fact, it requires that the feedback effects of each policymaker’s choice on future policy choices of the opponent be considered.

\(^9\) See also van Aarle et al. (1997). In their analysis of the dynamic interaction of monetary and fiscal authority in determining debt accumulation, which is in spirit very similar to ours, the authors use our same approach. As they recognise, the main advantage of the two-step procedure lies in its practicability, notwithstanding its limits.
The rest of the paper is organised as follows. The model is described in the next section and, in the appendix, it is shown how to derive the policymakers’ loss functions using, as an example, the well-known (micro-founded) model developed by Dixit and Lambertini (2003). In section 3 the second stage of the game is solved for the different institutional arrangements. In section 4 the outcomes of the different regimes are compared. In section 5 the choice of the institutional arrangement is considered (i.e. the first stage is solved). Section 6 concludes.

2. The Model

We consider an economy in which the central bank sets monetary policy and the government sets fiscal policy. Since this paper is not concerned with the issue of time inconsistency, it is assumed that there exists a mechanism to enforce the announced policies.

The government takes decisions about public expenditure and about the tax structure. The amount by which public expenditure exceeds tax revenues determines fiscal deficit (net of interest payments). The deficit is financed by issuing either interest bearing bonds or money.

The decision about the way in which public debt has to be financed is under the control of the central bank, which exercises this control through open market operations.

Each policymaker chooses its policy by minimising an explicit loss function under the constraints of the economic environment. In what follows we focus on the policymakers’ loss functions directly defined on their control variables and on public debt. The other final objectives of monetary and fiscal policy, which concern short-run stabilization, are implicit in the desired values of the policymakers’ control variables. These values depend,

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10 We believe that our results are more general than it may appear looking at the model developed in the appendix and that they are relevant for any situation in which two policymakers share a common goal and each policymaker, in trying to reach its other objective, has a negative impact on the common goal.

11 The private sector consists of small agents. They do not act strategically, and therefore, their joint decisions on consumption and savings affect only the parameters of the model.

12 It can be a legislative device or an external authority which prevents the players from reneging on the announcements made; a reputation mechanism, so that the player who deviates will not be believed in the future; an institution which enhance players credibility.
among other things, on the underlying structure of the economy and should vary with the information structure (which in our context is identified by the policy regime). In order to simplify our analysis and focus on the effects of monetary and fiscal policy interactions on public debt, we assume the unconstrained optimal strategies, i.e. those that would be chosen if public debt were not considered and monetary and fiscal policies only concern was short-run stabilisation, not to vary across policy regimes. This would for example be the case if there were complete separation of tasks between monetary and fiscal authorities and if a single target were assigned to each of them.

We show how to derive the optimal unconstraint policies in the appendix using, as an example, Dixit and Lambertini’s (2003) model but assuming that, in the short run, the central bank only aims at stabilising inflation while the fiscal authority’s only concern is to stabilise output. This assumption should be thought of as reflecting a policy assignment in which the central bank mandate is to stabilize inflation so that the monetary authority is not supposed to trade-off inflation against output and, because of the central bank mandate, the government is not concerned with price stability.

A complete micro-economic and micro-political foundation of the policymakers’ objectives is beyond the scope of this paper. What we have in mind is a situation in which institutions must receive political support and policy assignments reflect the interest of different groups in society. Thus, for example, a monetary policy devoted to price stability would be the result of the political power of interest groups averse to inflation (e.g. Miller, 1998; Harrendorf and Neumann, 2003; and Di Gioacchino et al. 2004).13

To prevent excessive debt accumulation, both policymakers, while targeting unconstrained strategies, aim at controlling public debt. This assumption is meant to capture the shared preoccupation that a broadly balanced policy mix emerges from the decentralized decisions of the government and the central bank.14

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13 This interpretation differs from the one which has become standard in the literature, where it is assumed that policymakers’ objectives differ from the preferences of the representative agent and, for reasons usually not discussed, the policy assignment under consideration is assumed to deliver greater welfare than purely representative authorities (e.g. Bayer, 1999; and Beetsma et al., 2001).

14 Tabellini (1987) justifies the inclusion of public debt in the policymakers loss function appealing to the fact that, in the absence of lump-sum taxes, a larger stock of public debt implies larger tax distortions in order to pay interest on the debt. Another reason for
Let $m(t)$ be the creation of monetary base against liability of the Treasure at time $t$, which is under the control of the central bank; $M$ be monetary policy unconstraint strategy, which reflects an inflation target, assumed constant over time; $f(t)$ be fiscal deficit (net of interest payments) at time $t$, which is under the control of the government; $F$ be fiscal policy unconstraint strategy, which reflects an output target, assumed constant over time; $d(t)$ be the stock of nominal public debt outstanding at the beginning of period $t$. All variables have been divided by nominal income.

The central bank chooses $m(t)$ to minimize a quadratic loss function given by a weighted sum of the deviations of $m(t)$ and $d(t)$ from their targets ($M$ and zero, respectively). The government chooses $f(t)$ to minimize a quadratic loss function given by a weighted sum of the deviations of $f(t)$ and $d(t)$ from their targets ($F$ and zero, respectively).15

The inter-temporal loss functions for the government and for the central bank are, respectively:16

$$
\begin{align*}
G(0) &= \int_{t=0}^{\infty} L_G(t) e^{-\lambda t} dt = \frac{1}{2} \int_{t=0}^{\infty} \left[ (f(t) - F)^2 + \alpha d(t)^2 \right] e^{-\lambda t} dt \\
V(0) &= \int_{t=0}^{\infty} L_V(t) e^{-\lambda t} dt = \frac{1}{2} \int_{t=0}^{\infty} \left[ (m(t) - M)^2 + \beta d(t)^2 \right] e^{-\lambda t} dt
\end{align*}
$$

where $L_G(t)$ and $L_V(t)$ are the instantaneous losses of the government and central bank, respectively. The parameters $\alpha$ and $\beta$ indicate the relative weight assigned to public debt by the two policymakers. If one of them did not care about public debt, it would set its control variable equal to its

including the level of public debt among the central bank’s objectives is offered by the fiscal theory of the price level. According to this theory, if fiscal policy does not ensure satisfaction of the government inter-temporal budget constraint, then the price level must do so and the central bank cannot control inflation (see Woodford, 2001 and literature cited therein). If doubts exist on the independence of the central bank, further reasons for including public debt among monetary policy targets are the preoccupation for an inflation bail-out, were the central bank forced to give way to government pressure for monetising the debt, or for an ex-post bail-out, in case of a financial crisis stemming from government defaulting on its debt.

15 Notice that both policymakers care about public debt and the common target for it is zero.

16 Notice that the central bank and the government have the same rate of time preference $\rho$ (assumed to be constant over time) and the same time horizon, which is infinite.
target, irrespectively of the other policymaker’s actions. The more a policymaker cares about debt, the more it reacts to a variation of the opponent’s control variable and the more fiscal policy and monetary policy are interdependent.

We do not explicitly consider society’s preferences. However, if the social loss function is a concave combination of the policymakers’ loss functions, any institutional arrangement that makes both policymakers better off with respect to a given one, represents a social welfare improvement. In the conclusions, we will exploit this hypothesis to discuss policy implication for institution design.

Each policymaker minimizes its loss function subject to the government dynamic budget constraint, which can be written as:

\[ \dot{d}(t) = -a d(t) + f(t) - m(t) \quad \text{with} \quad d(0) = d_0 \]

The parameter \( a \), which is the difference between the growth rate of real income and the real interest rate, is assumed to be constant.\(^{17}\) In order to eliminate explosive solutions and the problem of debt sustainability, we also assume that the growth rate of real income is larger than the real interest rate. In other words, since we are interested in the steady state values of the variables associated with different institutional setting, the assumptions regarding \( a \) simply isolate our objective from the effects of business cycle and debt sustainability, which are outside the scope of the present paper, notwithstanding their relevance.\(^{18}\)

Notice that if policymakers did not care about debt (i.e. if \( \alpha = 0 \) and \( \beta = 0 \)), then the steady state value for this variable would be \( \bar{d} = \frac{F - M}{a} \).\(^{19}\) By contrast, when policymakers take public debt into account, its steady state

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\(^{17}\) The assumption that \( a \) is constant over time is introduced to limit the analytical complexity. An endogenous real interest rate would imply a non-linear dynamic constraint in the differential game. Similar assumptions are very common in the literature, see, among others, Tabellini (1986), Jensen (1994), van Aarle et al. (1997), Beetsma and Bovenberg (1997), and Natale and Tirelli (2003).

\(^{18}\) As suggested by Darby (1984), if the growth rate of real income is larger than the real interest rate, then, contrary to Sargent and Wallace (1981), the economy is not on an explosive path and the “arithmetic is not so unpleasant.”

\(^{19}\) Without loss of generality, we only consider the case in which \( F > M \) so that \( \bar{d} > 0 \). See Appendix A,
value is always lower than $d$ and it depends on the relative importance of debt in the policymakers’ preferences (i.e. $\alpha$ and $\beta$) as well as on the institutional regime (information setting) in which the policymakers interact. In fact, if both policymakers target public debt, each of them can try to free-ride and leave onto the opponent the burden of reducing public debt, the more so when debt stabilization is costly in terms of instrumental variable adjustment and when a policymaker is able to pre-commit its policy.

In the next section we solve the (differential) game under different regimes and in section 4 we evaluate and compare the outcomes.

3. Monetary-Fiscal Interactions: equilibrium outcomes in different policy regimes

3.1 Simultaneous moves (Nash equilibrium)

Suppose that the central bank and the government, simultaneously, submit a plan of their future course of action and commit to carrying out this plan. The appropriate solution in this context is the so called open-loop Nash equilibrium, in which each player takes the current and future actions of the opponent as given. More precisely, a pair of strategies (one for each player) is an open-loop Nash equilibrium if, and only if, the time path of actions to which each player commits is an optimal response to the time path to which the opponent has committed.\(^{20}\)

Each policymaker’s optimal plan of actions is obtained by minimizing its loss function subject to the government’s budget constraint. The Present Value Hamiltonians for the two policymakers can be written as:

\[
H_G(t) = -\frac{1}{2} \left[ (f(t) - F)^2 + \alpha d(t)^2 \right] + \lambda_1 \left[ -ad(t) + f(t) - m(t) \right]
\]

\[
H_V(t) = -\frac{1}{2} \left[ (m(t) - M)^2 + \beta d(t)^2 \right] + \lambda_2 \left[ -ad(t) + f(t) - m(t) \right]
\]

and corresponding first order conditions are (in each instant of time):

\(^{20}\) Notice that in our framework open-loop solutions are not time inconsistent since first order conditions involve neither rival control variables nor state variables (cf. equations (6) and (7)). See Dockner et al. (2000: Chapters 5 and 7) for an extended discussion.
\( f(t) - F - \lambda_1(t) = 0 \)  
\( m(t) - M + \lambda_2(t) = 0 \)  
\( \dot{\lambda}_1(t) - \rho \lambda_1 = \alpha d(t) + a \dot{\lambda}_1 \)  
\( \dot{\lambda}_2(t) - \rho \lambda_2 = \beta d(t) + a \dot{\lambda}_2 \)

The adjoint equations (8) and (9) together with equation (3), after substituting equations (6) and (7), form a three differential equation system. The solution of this system with respect to the vector \( \{d(t), \dot{\lambda}_1(t), \dot{\lambda}_2(t)\} \) gives the Nash open-loop equilibrium. Notice that, from equations (6) and (7), Lagrange multipliers correspond to the policymakers target deviation costs (i.e. \( \lambda_1(t) = f(t) - F \) and \( \lambda_2(t) = M - m(t) \)). Hence, studying the dynamics of \( \{d(t), \dot{\lambda}_1(t), \dot{\lambda}_2(t)\} \) means studying the dynamics of policymakers deviations from their targets (recall that zero is their common debt target).

Steady state equilibrium target deviations for the debt and controls are easily found:

\[ t_N^d = \frac{\sigma}{\alpha + \beta + a \sigma} (F - M) \]

\[ t_N^{\Delta f} = F - f(t) = \frac{\alpha}{\alpha + \beta + a \sigma} (F - M) \]

\[ t_N^{\Delta m} = m(t) - M = \frac{\beta}{\alpha + \beta + a \sigma} (F - M) \]

where \( \sigma = \rho + a > 0 \). Since our purpose is to compare the policymakers’ losses in different regimes, we prefer to express the results in terms of deviations from targets. Steady-state fiscal deficit can be directly computed from equations (11). Monetary instrument can be obtained in an analogous manner.

\(^{21}\) The solutions of all the regimes can be found by applying the algorithm provided by Appendix B, which can also be used for simulation purposes.
Equations (11) and (12) measure how much policymakers are active in fiscal consolidation with respect to their alternative objective. In other words, equation (11) measures how much the government restricts its expenditure in order to obtain a public debt lower than the level consistent with its fiscal target. Equation (12) has a similar interpretation.\(^{22}\)

In the second stage of the game, it is assumed that policymakers minimize their inter-temporal losses so that only the eventual steady state level matters. This assumption is consistent with our focus on permanent effects of monetary-fiscal interaction.

According to equilibrium outcomes and equations (1) and (2), steady state optimal losses turn out to be:

\[
\bar{G}_N = -\frac{\alpha (\alpha + \sigma^2)}{2} \left( \frac{F - M}{\alpha + \beta + a\sigma} \right)^2
\]

\[
\bar{F}_N = -\frac{\beta (\beta + \sigma^2)}{2} \left( \frac{F - M}{\alpha + \beta + a\sigma} \right)^2
\]

Note that, if \(\alpha = \beta\) then \(G_N = V_N\), \(G_W = V_W\), \(G_G = V_G\), and \(G_F = V_F\), that is, the open-loop Nash solution gives equal payoff to symmetric players.

### 3.2 Leadership

Open-loop Nash equilibrium is the appropriate solution concept when players take their decisions simultaneously. If one player can take its decisions knowing the opponent’s reaction, such a player becomes the leader; the other player, who reacts (rationally) to the leader’s decision, is the follower. In solving its optimization problem, the leader, knowing the follower’s reaction function, takes it into account as an additional constraint.

The policy game can be solved with the government as leader and the central bank as follower (fiscal leadership regime) or vice versa.\(^{23}\)

\(^{22}\) Formally, this is due to the fact that, in all the regimes, deviations from instrumental variable targets equal the shadow prices of the fiscal debt for the policymakers (see equations (6) and (7)).

\(^{23}\) “...a monetary authority sufficiently powerful vis-a-vis the fiscal authority that by the imposition of slower rates of growth of base money [...] it can successfully constrain fiscal policy by telling the fiscal authority how much seigniorage it can expect. [...] On the other
The solution with the government as leader and the central bank as follower is found by solving the government optimization problem: minimize (1) subject to (3) and the central bank’s first order condition and co-state constraint (equations (7) and (9)). Thus the government’s Hamiltonian has to be rewritten as follows:

\( H_g(t) = -\frac{1}{2} \left[ (f(t) - F)^2 + \alpha d(t)^2 \right] + \lambda_1 \dot{d}(t) + \lambda_2 \ddot{d}(t) \)  \hspace{1cm} (15)

First order conditions, obtained by maximizing (15), are given by equations (7) and (9) plus (16) and (17) below:

\( \dot{\lambda}_1(t) - \rho \lambda_1(t) = \alpha d(t) + a \lambda_2(t) - \beta \lambda_3(t) \)  \hspace{1cm} (16)

\( \dot{\lambda}_2(t) - \rho \lambda_2(t) = -\lambda_1(t) - \lambda_3(t) \sigma \)  \hspace{1cm} (17)

By solving the differential system of equations (3), (9), (16), and (17) and using equations (6) and (7), we get the following equilibrium target deviation outcomes:

\( \bar{\Delta}_G = \frac{(\beta + a \sigma) \sigma}{(\beta + a \sigma)^2 + \alpha a \sigma} (F - M) \)  \hspace{1cm} (18)

\( \Delta \bar{G} = \frac{\alpha \sigma}{(\beta + a \sigma)^2 + \alpha a \sigma} (F - M) \)  \hspace{1cm} (19)

\( \Delta \bar{m}_G = \frac{\beta (\beta + a \sigma)}{(\beta + a \sigma)^2 + \alpha a \sigma} (F - M) \)  \hspace{1cm} (20)

The steady state losses associated with fiscal leadership can be computed to be:

\( \bar{G}_G = -\frac{\alpha}{2} \left[ (\beta + a \sigma)^2 + \alpha a \sigma \right] \left[ \frac{\sigma (F - M)}{\left( \beta + a \sigma \right)^2 + \alpha a \sigma} \right]^2 \)  \hspace{1cm} (21)

hand, [...] the monetary authority is not in a position to influence the government’s deficit path but is limited simply to managing the debt that is implied by the deficit path chosen by the fiscal authority. Under this second scheme the monetary authority is much less powerful than under the first scheme” (Sargent and Wallace, 1981: 158).
Analogously, if the central bank leads and the government follows (monetary leadership), the solution is found by minimizing (2) subject to (3) and the government’s first order condition and co-state constraint (equations (6) and (9)). Maximization of the Hamiltonian yields:

\[
\dot{\lambda}_2(t) - \rho \lambda_4(t) = \beta d(t) + a \lambda_2(t) - \alpha \lambda_4(t)
\]

\[
\dot{\lambda}_4(t) - \rho \lambda_4(t) = -\lambda_2(t) - \lambda_4(t)\sigma
\]

The steady state target deviations are found to be:

\[
\bar{d}_v = \frac{(\alpha + a\sigma)\sigma}{(\alpha + a\sigma)^2 + \beta a\sigma}(F - M)
\]

\[
\Delta \bar{y}_v = \frac{\alpha(\alpha + a\sigma)}{(\alpha + a\sigma)^2 + a\beta\sigma}(F - M)
\]

\[
\Delta \bar{m}_v = -\frac{\beta\sigma a}{(\alpha + a\sigma)^2 + a\beta\sigma}(F - M)
\]

The losses associated with monetary leadership are:

\[
\bar{G}_v = -\frac{\alpha(\alpha + \sigma^2)}{2} \left[ \frac{(\alpha + a\sigma)(F - M)}{(\alpha + a\sigma)^2 + \beta a\sigma} \right]^2
\]

\[
\bar{V}_v = -\frac{\beta \left[ (\alpha + a\sigma)^2 + \beta a^2 \right]}{2} \left[ \frac{\sigma(F - M)}{(\alpha + a\sigma)^2 + \beta \sigma} \right]^2
\]

### 3.3 The warfare regime

A leadership equilibrium requires that the players have reached an agreement about who is going to lead the game and who is going to follow. If the players do not coordinate their actions, it may well be that each of them tries to act as leader. In this case the first order conditions for an optimum are given by equations (16), (17), (23), and (24) together with
equations (3), (6), and (7). The outcome associated with this equilibrium is
the so-called (Stackelberg) warfare. Steady state outcomes associated with
the warfare regime turn out to be:

\[
\begin{align*}
(29) \quad \tilde{d}_w &= \frac{(\alpha + a\sigma)(\beta + a\sigma)}{a(\alpha + \beta + a\sigma)^2 - \alpha\beta}(F - M) \\
(30) \quad \Delta \tilde{f}_w &= \frac{\alpha(\alpha + a\sigma)}{(\alpha + \beta + a\sigma)^2 - \alpha\beta}(F - M) \\
(31) \quad \Delta \tilde{m}_w &= \frac{\beta(\beta + a\sigma)}{(\alpha + \beta + a\sigma)^2 - \alpha\beta}(F - M)
\end{align*}
\]

The steady state losses associated with the warfare regime can be computed
to be:

\[
\begin{align*}
(32) \quad \tilde{G}_w &= -\frac{\alpha}{2} \left[ \alpha a^2 + (\beta + a\sigma)^2 \right] \left[ \frac{(\alpha + a\sigma)(F - M)}{a(\alpha + \beta + a\sigma)^2 - \alpha\beta} \right]^2 \\
(33) \quad \tilde{V}_w &= -\frac{\beta}{2} \left[ (\alpha + a\sigma)^2 + \beta a^2 \right] \left[ \frac{(\beta + a\sigma)(F - M)}{(\alpha + \beta + a\sigma)^2 - \alpha\beta} \right]^2
\end{align*}
\]

4. Evaluation of the outcomes

In the game we have been describing both policymakers aim at stabilizing
the debt, but each of them prefers that the other one takes the burden of
doing so. In the Nash regime each policymaker takes as given the policy of
the other. In this regime, steady state debt decreases in the relative weight
assigned to this variable by the policymakers. Differently, in a leadership
regime, the leader exploits the first-mover advantage to pre-commit to a low
level of debt stabilization and to impose a greater responsibility towards this
objective onto the other policymaker. In this regime, decreasing the weight
that the follower attaches to the common goal of public debt stabilization
reduces the leader’s incentive to place a bigger burden of the adjustment
onto the opponent. Therefore, the debt associated with leadership by the policymaker who cares relatively more about debt is lower than that associated with leadership by the other policymaker. A comparison of the warfare with the other regimes shows that steady state debt is higher under warfare than under any of the other regimes. In the warfare regime, in fact, each policymaker tries to shift the burden of debt stabilization onto the other, and for the same parameters’ values, each player puts less effort in debt stabilization. As a result of this free-riding policy, debt stabilization is reduced. Moreover, an increase in the relative importance of debt for one policymaker reduces its incentive to shift the debt burden on the opponent but it raises the other policymaker’s incentive to do so. Therefore, if the last effect prevails on the first, increased preferences for debt stabilization might, paradoxically, result in higher stocks of debt. Since the ex ante policymakers’ conjecture about the rival’s behaviour will never be realized ex post, the debt will always result higher than expected. The above discussion implies that a policymaker’s instrument is closer to its target the less the policymaker stabilizes the debt, either because it doesn’t care about debt or because being the leader, it can impose debt stabilization onto the opponent. In this respect, it can be verified that in a leadership regime, the leader’s (follower’s) control variable is closer to (more distant from) its target than it is under the Nash regime and that instrument’s deviations from target increase in the relative weight assigned to public debt by the policymakers.

In order to evaluate the losses associated with each regime we also need do compare instruments’ deviation from target across outcomes. In general, the leader’s instrument deviation from its target is smaller than the follower’s. However, target deviations also depend on the other policymaker’s strategy: the deviation is smaller (higher) when the opponent plays follower (leader). Hence, the leader’s (follower’s) instrument, in a leadership regime, is the closest (farthest) from its target. Instruments’ deviations in the warfare and in the Nash regimes are in between those two extreme cases and lower in the warfare regime. The intuition for this ranking straightforwardly follows from our previous discussion on the policymakers’ incentive to free-ride in the different regimes.
We can summarize the above discussion as follows:\textsuperscript{24}

1. Debt in either of the Stackelberg regimes is higher than in the Nash regime but lower than in the warfare regime, i.e. $d_N < d_V$ ($d_N < d_G$) and $d_V < d_W$ ($d_V < d_G$).

2. If the central bank cares about public debt more than the government does ($\beta > \alpha$), then debt under fiscal leadership is higher than under monetary leadership ($d_G > d_V$) and vice versa for $\alpha > \beta$.

3. A policymaker’s instrument deviation from its target is at its minimum (maximum) value when the policymaker is leader (follower) and it is lower in the warfare regime than in the Nash one, i.e. $\Delta f_G < \Delta f_W < \Delta f_N < \Delta f_V$ and $\Delta m_V < \Delta m_W < \Delta m_N < \Delta m_G$.

From observation 2 it follows that:

a) If a policymaker cares about debt relatively more than the opponent, then he rather be leader than follower. Note that the contrary is not necessarily true.

From observations 1 and 3 it follows that:

b) For each policymaker, losses in the Nash regime are lower than those suffered when being follower, i.e. $V_N < V_G$ and $G_N < G_V$.

Moreover, the leader’s losses are lower than those in the warfare regime, i.e. $V_V < V_W$ and $G_G < G_W$.

In principle, it would be possible for a policymaker to prefer being the follower than the leader. This would be the case if the gains in terms of debt deviation were to compensate the losses in terms of instrument deviation.

The above inequalities do not allow a unique ranking of losses. However, assuming that the policymakers have the same ordering of preferences the restrictions imposed by (b) above reduce the number of possible rankings so that only six of them are possible.\textsuperscript{25} In the next section we analyze those

\textsuperscript{24} The following results can be easily verified with standard calculus. Proofs are however available upon request.

\textsuperscript{25} The six rankings are obtained by combining the inequalities $V_V < V_W$ and $G_G < G_W$. 

16
possibilities and conclude that one of them is the most interesting for the purpose of the present paper.

5. Institutional Design

In this section we solve the first stage of the game in which the institutional regime is chosen. At this stage, each policymaker can choose to be the leader (l) or to follow the leader (f) and the game payoffs are the equilibrium losses of the second stage. The matrix below represents the government’s and central bank’s losses under the regimes discussed in the previous sections. If the policymakers choose complementary roles, one of the leadership equilibria emerges. If both choose to lead, we have the warfare. By contrast, if both policymakers choose to play as followers, the Nash solution results.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>l</th>
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<tbody>
<tr>
<td>BC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>(\bar{V}_N, \bar{G}_N)</td>
<td>(\bar{V}_G, \bar{G}_G)</td>
</tr>
<tr>
<td>l</td>
<td>(\bar{V}_P, \bar{G}_P)</td>
<td>(\bar{V}_W, \bar{G}_W)</td>
</tr>
</tbody>
</table>

We concluded the previous section arguing that if the policymakers have the same ordering of preferences, then only six rankings of losses are possible. Of these rankings, four describe a game with a unique equilibrium, namely the Nash regime. The other two describe a coordination game, i.e. a game with multiple equilibria. In these games, the players’ problem is to coordinate their decisions and reach one of the equilibria. A coordination game is with common interest if the equilibria are Pareto-rankable; in this case it is reasonable to expect that the policymakers coordinate and play the

---

26 These are, together with their homologues for G (recall that we have assumed the same ordering of the losses between the two policymakers): 1) \( \bar{V}_N < \bar{V}_G < \bar{V}_W \), 2) \( \bar{V}_N < \bar{V}_G < \bar{V}_W \), 3) \( \bar{V}_N < \bar{V}_G < \bar{V}_W \), 4) \( \bar{V}_N < \bar{V}_G < \bar{V}_W \)
Pareto-efficient equilibrium. Coordination games without common interest are more problematic, since players prefer different equilibria. In our context, this possibility occurs under the ranking $V_F < V_N < V_G < V_W$ and $G_G < G_N < G_F < G_W$. In this case both leadership regimes are equilibria of the game. However, each policymaker prefers a different solution (i.e. each policymakers prefers to be the leader). Henceforth, we focus on this ranking which generates a conflict for the choice of the institutional arrangement.

In the context of monetary-fiscal interaction, the possibility of pre-play communication to reach an agreement appears to be a natural way to solve the conflict over the institutional arrangement. In this case, it is possible for the players to reach the so-called correlated equilibrium. Unlike mixed strategy Nash equilibrium, in a correlated equilibrium the players can coordinate their randomization.

If mixed-strategy Nash equilibrium were the solution of the first stage, there would be a positive probability for the warfare to be the outcome (see the table below which reports the probability of each outcome when the mixed-strategy Nash equilibrium is played).

Both players, instead, would prefer the warfare to have a zero probability of being the outcome. However, if the players randomize independently, there is no way in which this can be achieved. On the other hand, if the players could coordinate their randomization, the warfare would be excluded from the possible outcomes.

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27 In our context the ranking $V_S < V_F < V_W$ (together with its homologous for $G$) describes a coordination game with common interest.

28 Since binding agreements in a non-cooperative game are ruled out by definition, only self-enforcing agreements have to be considered.

29 The Nash equilibrium in mixed strategies is $\begin{bmatrix} p' & 1-p \\ p & 1-p \end{bmatrix}$. In this equilibrium the central bank must be indifferent between leader and follower. This requires that the probability with which the government chooses to follow the leader, denoted by $p$, should be such that: $V(1,.) = pV_f + (1-p)V_w = pV_f + (1-p)\bar{V}_f = V(.,1)$ Analogously, the probability with which the central bank chooses to follow the leader, denoted by $p'$, should be such that: $G(.,f) = p\bar{G}_f + (1-p)\bar{G}_w = p\bar{G}_f + (1-p')\bar{G}_w = G(.,1)$. 

---
Aumann (1974 and 1987) introduced the concept of correlated equilibrium, as an extension of Nash equilibrium, to allow for correlation between the players’ randomizations. A correlated strategy is a function \( f \) from a finite probability space into the space of actions (it is a random variable whose values are pairs of actions). As in the case of mixed strategies, the players base their choices on the observation of a random event but, unlike mixed strategies, with correlated strategies the observations are not independent. The easiest way to think of it is to suppose that there is an external referee who, after having observed the random event, suggests to each player which action he should take. If the players follow the suggestion, then the result is a correlated strategy. A correlated equilibrium is a correlated strategy which is a best response against the equilibrium strategy of the opponent and therefore it is self-enforcing. The distribution of a correlated strategy is the function that assigns to each pair of players’ actions \( (a_1, a_2) \) the value \( \text{prob}\{f^{-1}(a_1, a_2)\} \).

**Proposition:** In the game for the choice of the institutional regime any distribution:

<table>
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<tr>
<th>( G )</th>
<th>( f )</th>
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<tbody>
<tr>
<td>( BC )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f )</td>
<td>( p'p )</td>
<td>( p'(1-p) )</td>
</tr>
<tr>
<td>( l )</td>
<td>( p(1-p') )</td>
<td>( (1-p')(1-p) )</td>
</tr>
</tbody>
</table>

is the distribution of a correlated equilibrium for with \( q \leq \bar{q} \) where, as shown below, \( \bar{q} \) depends on the equilibrium losses in the second stage.
Proof. Suppose that the central bank has been told to play \( f \); it then knows that the government has been told to play either \( f \) (with probability \( \frac{1 - 2q}{1 - q} \)) or \( l \) (with probability \( \frac{q}{1 - q} \)). By following the suggestion and playing \( f \) the central bank’s loss is \( V(f, \cdot) = \frac{(1 - 2q)V_N + qV_G}{1 - q} \). By playing \( l \), instead, the central bank’s loss is \( V(l, \cdot) = \frac{(1 - 2q)V_f + qV_w}{1 - q} \). It follows that for any 
\[ q < \bar{q}_V = \frac{V_f - V_N}{V_f - V_w + 2(V_f - V_N)} \] the central bank is better off by following the suggestion. Analogously, for any 
\[ q < \bar{q}_G = \frac{G_f - G_N}{G_f - G_w + 2(G_f - G_N)} \] the government is better off by following the suggestion and play \( f \). Let 
\[ \bar{q} = \min \left\{ \bar{q}_V, \bar{q}_G, \frac{1}{2} \right\} \] then for any \( q \leq \bar{q} \) a correlated equilibrium is obtained (\( q \leq \frac{1}{2} \) is required for \( 1 - 2q \) to be non-negative).

6. Conclusions

The recognition that economic policy is not run by a unique policymaker and that different authorities take fiscal and monetary decisions makes the issue of how they coordinate their actions relevant. The outcome of monetary-fiscal interaction is determined by the rules of the game, i.e. the institutional regime in which the policymakers operate. Unlike most of the literature that studies monetary-fiscal interaction, our concern is not with short run macroeconomic stabilization; instead, we focus on the long run effects of institutional arrangements on public debt dynamics.

In this paper the interaction between fiscal and monetary policies has been analyzed as a two-stage game played by the central bank and the government. In the first stage the institutional regime is established. This determines the rules of the game, i.e. the equilibrium solution to be applied.
in the second stage, in which the policymakers minimize their loss functions subject to the government’s budget constraints. Since the policymakers have conflicting preferences over the institutional arrangement, the second stage presents a multiplicity of equilibria. Pre-play communication, to reach an agreement, allows to obtain the correlated equilibrium and to avoid the Pareto inferior outcome. In the correlated equilibrium monetary and fiscal leadership are both played. Under monetary leadership, the central bank is given first-mover advantage and it is not forced to “bail-out” fiscal decisions. Under fiscal leadership, the government is given first-mover advantage and the central bank is forced to monetize public debt and it cannot guarantee monetary stability.

It has been recognized that the government will try to influence monetary policy even in circumstances in which monetary financing of public debt by the central bank is prohibited by statute. We can therefore interpret our solution as describing an institutional setting in which either monetary or fiscal leadership emerge depending on whether the central bank is able or not to resist the government pressure.

If the social loss function is a concave combination of the policymakers’ loss functions, the design of institutions which favours the exchange of information between policy-makers, yet preserving a clear allocation of responsibilities, must be regarded as welfare improving.

This matter is of some relevance for the EMU. In fact, the Macroeconomic Dialogue (also known as the Cologne Process) provides an institutional forum for discussing economic policy. This we plan to investigate in a future extension of our paper including more independent fiscal authorities and one common central bank. In addition, we aim to study different information setting as feedback and Markov solutions and more deeply their dynamics.

Appendix A – Derivation of the Model: An Example

As suggested by Tabellini (1986 applying a two-step procedure that separates the effects of monetary-fiscal interactions on public debt from those on output and inflation allows to greatly simplify the dynamics of the

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30 See, e.g., the discussion in Dixit (2000).
31 Which is of major importance to ensure accountability of individual policy-makers
model. The first step of the procedure consists of computing the optimal policies in the absence of public debt. These unconstrained strategies have to be modified to take into account the effects on public debt dynamics. The adoption of this procedure allows us to write down a model in which the policymakers’ loss functions are directly defined on the policy instruments and on the state variable (public debt).

To derive the reduced-form model used in Section 2, we use, as an example, the well-known micro-founded model of Dixit and Lambertini (2003). However, the same reduced form can be derived from many different models describing the interaction between fiscal and monetary authorities.

Dixit and Lambertini’s (2003) framework can be summarised by four equations:

(a1) \[ L_f = \frac{1}{2} \left[ \left( x - x_f \right)^2 + \theta_f \left( \pi - \pi_f \right)^2 \right] \]

(a2) \[ L_m = \frac{1}{2} \left[ \theta_m \left( x - x_m \right)^2 + \left( \pi - \pi_m \right)^2 \right] \]

(a3) \[ x = \alpha g + b \left( \pi - \pi^e \right) \]

(a4) \[ \pi = \pi_0 + cg \]

where \( x = y - \bar{y} \) is the real output gap, \( \pi \) and \( \pi^e \) are inflation and expected inflation. Equations (a1) and (a2) describe the preferences of the fiscal and monetary authorities, respectively. Fiscal and monetary authorities set \( g \) and \( \pi_0 \). Equations (a3) and (a4) describe the real output gap and the inflation, which depends on policymakers’ choice (for more details see Dixit and Lambertini, 2003).

By solving the game, we obtain the following reaction functions:

(a4) \[ g = \frac{\left( \alpha + bc \right)b\pi^e - \left( b^2c + \theta_f \alpha + \alpha b \right)\pi_0 + \left( \alpha + bc \right)x_f + \theta_f c\pi_f}{\left( \alpha + bc \right)^2 + \theta_f \alpha^2} \]

(a5) \[ \pi_0 = \frac{\theta_m b^2\pi^e - \left( c + \theta_m b^2c + \theta_m \alpha b \right)g + \theta_m b x_m + \pi_m}{1 + \theta_m b^2} \]
The Nash solution, assuming rational expectations, is:

\[
\begin{align*}
    (a6) \quad g^* &= \frac{\theta_f \theta_m b c x_f - (\alpha + bc) x_f - \theta_f c (\pi_f - \pi_m)}{\alpha \theta_f \theta_m b c - \alpha - bc} \\
    (a7) \quad \pi_0^* &= \frac{\theta_f c^2 + \alpha \theta_f b c + \alpha^2}{\alpha (\theta_f \theta_m b c - \alpha - bc)} b \theta_m x_f - \frac{\alpha \theta_m b^2 c + \alpha^2 \theta_m b + bc^2 + \alpha c}{\alpha (\theta_f \theta_m b c - \alpha - bc)} x_m + \\
    &\quad + \frac{(c + \alpha \theta_f b) \theta_f c \pi_m - (\theta_f c^2 + abc + \alpha^2) \pi_f}{\alpha (\theta_f \theta_m b c - \alpha - bc)} \\
\end{align*}
\]

As compared with the model of Section 2, \( g^* \) and \( \pi_0^* \) correspond to \( F \) and \( M \), respectively. They can be interpreted as the unconstrained policies that would be chosen by the policymakers if they did not care about public debt.

By assuming an extreme assignment, in which the central bank is only concerned about inflation stabilization (i.e. \( \theta_m = 0 \)) and the government aims at stabilising output (i.e. \( \theta_f = 0 \)), we can parameterize the unconstrained policies as follows:

\[
\begin{align*}
    (a6) \quad F &= g^* = \frac{x_f}{\alpha} \\
    (a7) \quad M &= \pi_0^* = \pi_m - \frac{c}{\alpha} x_f \\
\end{align*}
\]

In equations (a6) and (a7), if \( \pi_m \) (\( x_f \)) is sufficiently small (large), then \( F > M \), which always holds if \( \pi_m = 0 \) and \( x_f > 0 \) (as it is often assumed).

By contrast, the non-economically interesting case \( M > F \) (negative debt bias) holds only for \( \pi_m > \left(1 + \frac{c}{\alpha}\right) x_f \).

It can be verified that equations (a6) and (a7) are independent of the regime. In fact, equations (a6) and (a7) also solve the Stackelberg and warfare games.
Appendix B – A Simple (MatLab) Algorithm to Compute Solutions

% parameters
alpha=0.5; beta=2; a=0.9; r=0.6; sigma=a+r;
% foc matrices
MN = [a  -1  -1; alpha sigma 0; beta 0 sigma];
MG = [a, -1, -1, 0; -alpha, -sigma, 0, beta; -beta, 0, -sigma, 0; 0, 1, 0, a];
MV = [a, -1, -1, 0; -alpha, -sigma, 0, 0; -beta, 0, -sigma, alpha; 0,0,1,a];
MW = [a, -1, -1, 0, 0; -alpha, -sigma, 0, beta, 0; -beta, 0, -sigma, 0, alpha; 0, 1, 0, a, 0; 0, 0, 1, 0, a];
% steady state solutions
N = -inv(MN)*[1,0,0]';
G = -inv(MG)*[1,0,0,0]';
V = -inv(MV)*[1,0,0,0]';
W = -inv(MW)*[1,0,0,0,0]';
% losses
GN=alpha*(N(1))^2+(N(2))^2;
GG=alpha*(G(1))^2+(G(2))^2;
GV=alpha*(V(1))^2+(V(2))^2;
GW=alpha*(W(1))^2+(W(2))^2;
VN=beta*(N(1))^2+(N(3))^2;
VG=beta*(G(1))^2+(G(3))^2;
VV=beta*(V(1))^2+(V(3))^2;
VW=beta*(W(1))^2+(W(3))^2;
% outcome matrix
Game=[VN,GN,VG,GG;VV,GV,VW,VW];

References


