Comments Welcomed

Why Is It So Hard To Measure the Current Output Gap?

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Abstract:

In order for time series estimates of the output gap to be useful to policy makers, this paper argues that two factors will be critical. First, they must be able to produce an estimate of the *current* output gap based only on past information. Put another way, to evaluate the performance of such estimators, we should focus on their properties as *filters* rather than *smoothers*. Second, the decomposition of actual output into potential output and the output gap must not be based on arbitrary assumptions on the time-series behaviour of these variables. In general, this leads to a multiplicity of possible solutions that may differ greatly in their policy implications. The arbitrary selection of one of these creates arbitrary policy advice.

A trivial but flexible estimator of the output gap (dubbed TOFU) can be constructed which satisfies both of the above criteria. It estimates the output gap as part of a larger economic relationship. A discussion of its properties shows that except under very strict circumstances, such time series methods will produce better estimates of output gaps *ex post* than they will *ex ante*. Furthermore, under certain conditions, contemporary and historical data will be of no use in estimating the output gap whatsoever. This in turn places limits on the extent to which time series methods can hope to improve upon structural estimates of the output gap.

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I. Introduction

It has been known for some time that estimates of potential output (and the related concept of NAIRU) vary widely across methods and researchers, and that some of these estimates have quite wide confidence intervals. Setterfield *et al.* (1992) generated NAIRU estimates for prime-age males that covered virtually the entire range of their actual unemployment rate for the 1956-87 period. Confidence intervals for the US NAIRU estimated by Weiner (1993) similarly imply that the actual unemployment rate was never *significantly* above or below the NAIRU during the 1962-1992 period.[1] Rose's (1988) survey concluded that ". . . [Our] estimates range widely and, in general, are not statistically well determined. . . . the same uncertainty surrounds the point estimates presented by other researchers."[2]

A number of authors have suggested that while structural estimates of potential output or NAIRU do not produce very robust results, the addition of time-series techniques may help to improve this uncertainty.[3] This paper examines the extent to which such techniques may be expected to improve a policy maker's estimates of the output gap.

The first section reviews the distinction between the problem of detrending output for business cycle research, and that of estimating the current output gap. Standard detrending methods (on which there has been considerable research) are shown to be ill-suited to the policy-maker's needs. The following section then considers the effects of alternative identifying assumptions on the robustness of the resulting estimates of the output gap. This in turn suggests that more theory is required to ensure that estimates will be meaningful. The third section describes an alternative way of estimating the output gap which takes account of the criticisms introduced in the previous sections. The fourth section then discusses what appears to be a previously unexplored link between the usefulness of time series methods and the economic mechanism causing the business cycle. In particular, there are some cases where time-series techniques will be completely uninformative.

1. This claim is based on calculations provided by the Economic Research Department of the Federal Reserve Bank of Kansas City. The author would like to thank Timothy Schmidt in particular for his assistance.

2. Rose (1988) p. *i*.

3. For example, see Laxton and Tetlow (1992).

II. Filtering and Smoothing

There is a considerable literature which estimates output gaps (or the related concepts of a "natural" or "non-accelerating inflation" rate of unemployment) from structural models. A key advantage of this approach is that they may allow one to construct estimates of the output gap which are invariant to changes in macroeconomic policy; that is, they satisfy the Lucas Critique (1976). The time series methods that are discussed below will make use of non-structural models of the time-series properties of macroeconomic data, and so cannot reasonably be expected to give policy-invariant estimates. Why then would policy makers be interested in the application of time-series methods to this question?

One explanation is that despite the considerable resources that have been devoted to structural estimates, the accuracy of such estimates is far from satisfactory. It is therefore natural that alternatives to structural estimation would eventually be explored. Another explanation is that even policy-sensitive estimates of output gaps can be valuable in a policy-making context. In particular, they may provide valuable information for the conduct of policy *within* a given policy regime. For example, within a regime of targeting inflation, such estimates might provide useful information on whether the monetary stance should be loosened or tightened to meet the target. Whatever the reason, the fact is that policy-making institutions are actively investigating the application of time series methods as an alternative or adjunct to purely structural estimates. Most of the work that has been done to date on this topic has been done in affiliation with a policy-making body.[4]

Of course, considerable work on the time series analysis of output has already been done in a non-policy context. The relevant literature can be grouped into two main fields. One of these considers various ways in which we can decompose output (or more generally, realizations of any univariate nonstationary stochastic data-generating process) into a nonstationary trend and a stationary cyclical component. The other field has concentrated on the question of how data should be detrended in order to characterize the business cycle. However, the policy maker's problem is different enough from both of these problems that it requires a new approach. The discussion of trend/cycle decompositions will be deferred until the next section, and the remainder of this section will discuss the problem of detrending.

Detrending is typically used as a means of making business cycle data stationary prior to examining the ability of a (stationary) business cycle model to capture important features of the data. This differs from the policy maker's problem of measuring the output gap in four important respects.

First, it is not enough for the policy maker to select *any* stationary component from the data; instead, he is trying to isolate a stationary component with a particular economic interpretation. As will be discussed in greater detail in the next section, many different stationary components can be extracted, but these may have widely differing economic interpretations.

Second, in order to test a business cycle model, the researcher usually limits the applica-

4. For example, Kuttner (1992) was published by the Federal Reserve Bank of Chicago, Laxton and Tetlow (1992) was published by the Bank of Canada, Quah and Vahey (1995) was written at the Bank of England and Giorno, et al. (1995) was published by the OECD.

tion of economic theory to the construction of the model. The data is then made stationary in an atheoretical way in the hope that this will serve as a check on the theory's adequacy. The policy maker is not seeking to test a theory, but to estimate a variable that is not directly observable. It therefore makes sense for the policy maker use an economic model to assist in detrending.[5]

Third, a researcher typically treats the detrended data as given, whereas the policy maker may treat the detrended data as an estimated series that is subject to estimation error. In particular, the latter may wish to determine the probability that the output gap is positive (or negative) at a given point in time. More generally, greater uncertainty in the estimate of the output gap could be used to justify a policy that is less responsive to the estimated gap.

Fourth, given a data set that ends at time *T,* the policy maker is most interested in estimating the output gap at the end of that sample, whereas the researcher may be content to have detrended data only for some subsample that does not include the last few observations. In the time series jargon, the researcher is typically interested in the application of a *smoother* (i.e. a function which may use observations from before and after *t* in order to estimate some quantity at *t*.) The policy maker's problem is principally one of *filtering* (using only information available by *t* to estimate a quantity at *t*.)

These differences can be seen more clearly if we consider some of the methods suggested for detrending data. Three good examples of these are the first-difference filter, the Hodrick-Prescott filter, and the Band-Pass filter.[6] All will produce a stationary component from an I(1) or linearly trending series. However, these stationary components may differ considerably among themselves, and the results of the Hodrick-Prescott and Band-Pass filters will in turn depend on the exact parameters used. Choosing one of these as an estimate of the output gap has traditionally relied on visually inspecting the results to see whether they are consistent with one's preconceptions rather than on the use of any objective yardstick. Theory is rarely used explicitly to justify the choice of one such measure over another.

The application of the Hodrick-Prescott (HP) and Band-Pass (BP) filters is problematic since these are actually *smoothers* rather than *filters* in the terminology introduced above. The BP "filter" is simply the difference between the original series and a symmetric weighted average of itself. The symmetry restriction implies that the smoother cannot produce estimates near the beginning or the end of the sample. The HP "filter" is symmetric in the middle of the sample, but gradually becomes a one-sided weighted average as one approaches the beginning or end of a sample. This implies that it behaves as a true filter at the last observation of a sample.

Figure 1 compares the weight placed on each observation at various points in the sample for the HP *smoother* used on quarterly data from 1947Q1 to 1994Q3. For example, observation 96 shows us that in the centre of our data sample, the smoother appears perfectly symmetric. However, as we move towards the end of the data, the smoother starts to become truncated as some leads of the series are no longer available. At four years from the end of the sample (Obs. 16), the smoother is still quite smooth and symmetric. Two years from the end (Obs. 8), we start to see an abrupt truncation at the end of the sample. In the last year (Obs. 1-4), the cutoff becomes more

5. Gregory and Smith (1993) and Smith (1994) pursue this idea in the context of detrending.

6. The Band Pass filter was recently proposed by Baxter and King (1995).

pronounced and the degree of asymmetry is heightened. Since the smoother converges to the filter at the end of the sample, the curve for Obs. 1 shows us the weights used by the true HP *filter.*

This difference in weights has two important effects. The first effect is that the true filter puts most of its weight on a very small number of observations. This means that its estimates of deviations from trend will tend to be more variable than those of the smoother and more sensitive to minor revisions in a few data points. The second effect comes from the lack of symmetry, which implies that filtered estimates will tend to lag changes in trend. There is an obvious trade-off between these two effects; the less the filter concentrates its weight on the most recent observations, the greater the lag that it introduces.

One way to measure the extent of the second effect is to calculate the filter's lag. The median lag at observation *t* is calculated by finding the largest *k* such that at least half of the filter's weight is placed on observations at or before *t-k*. Another measure is the mean lag, which for observation *t* is given by

$$\sum_{k=1}^{191} \alpha_{t,k} \cdot (k-t) \qquad \text{(EQ 1)}$$

where $\alpha_{t,k}$ is the weight that the trend at time *t* puts on the observation at time *k*. As shown on Figure 1, the median lag of the filter in this sample is only 2 quarters. However, because of the highly skewed distribution of these weights, the mean lags are much larger; almost 9 quarters.

Figure 2 graphs the smoothed and filtered estimates of deviations from trend for the log of US real GNP using quarterly data for period mentioned above.[7] (Positive numbers imply that actual GNP is below trend.) Both series have approximately the same mean and standard deviation, and the two series are positively correlated. The true HP filter gives slightly larger peaks and troughs, while the smoother seems to lead the true filter by a few quarters . Figure 3 graphs the difference between these two series (shown as a fraction of actual GNP.) Differences are often 2% of GNP or more, such as in 1950, 1954, 1964, 1977, 1980, 1983-84, etc.

These results suggest that the difference between using the Hodrick-Prescott method of detrending as a *smoother* and as a *filter* may be economically significant. These differences would be larger still if the true filter were run on preliminary announcements of data (as a policymaker would) rather than on the subsequently revised estimates. However, the use of the HP detrending method is invariably based on a visual inspection of the *smoothed* series rather than the *filtered* series. This in turn suggests a confusion between the policy maker's problem and that of simply detrending data.

7. These estimates were constructed using a smoothing parameter of 1600.

III. Identification and Misspecification

The previous section discussed some of the differences between the problem of simply detrending data and the policymaker's problem in estimating an output gap. In this section, we review lessons from the literature on trend/cycle decompositions.

This literature examines solutions to the problem of decomposing a single nonstationary time series y_t into the sum of a nonstationary component (the trend) and a stationary component (the cycle). A number of alternative decompositions have been proposed. Perhaps the most popular and influential of these was the Beveridge-Nelson (1981) decomposition, which specified that the nonstationary component follow a random walk. A number of authors attempted to draw conclusions about the relative importance or real and nominal shocks from these decompositions, associating the "trend" component with "real" macroeconomic shocks and the cyclic component with "nominal" shocks. This literature was strongly criticised by Quah (1992), who notes that there are an infinite number of ways in which to define the cycle-trend decomposition of a series. Depending on the decomposition that we choose, we can make the fraction of the variance due to the cyclic component in a finite sample arbitrarily close to 100%, while the assumption that the trend is a random walk will maximize the trend's share of the variance.

Quah's critique highlights an important fact. There are an infinite number of ways in which to define the cyclic component of a series, and the definition that we choose can make an important difference to the properties of that cyclic component. To understand the extent of this problem, consider the following. For *any* nonstationary series y_t, we can subtract from it *any* stationary series c_t to define a trend τ_t. Since c_t and τ_t will sum to y_t, *any* stationary series can therefore be defined to the cyclic component of y_t. Since shifting a stationary series by any number of leads or lags still leaves the series stationary, this means that we can similarly transform any cyclic component to define another measure of the cycle. Since multiplying a stationary series by any scalar constant leaves a stationary series, we can scale a cyclic component by any number to define another measure of the cycle. Clearly, the problem is not that is difficult to construct a cyclic component. The problem is that with so many possible definitions, it is difficult to select a unique and meaningful one.

This problem of non-uniqueness is directly applicable to the problem of estimating the output gap. If we define the output gap to be just the stationary component of output, this leaves so many possible definitions that the results are of no use in guiding policy. As a simple example, for any possible measure of the output gap X, $-X$ and $0 \cdot X$ would also be reasonable candidates, although their implications for policy would be very different.

To arrive at a more restrictive definition of the output gap, a broad range of identifying assumptions have been suggested. While each produces a unique measure of the gap, these measures can only be as economically meaningful as their identifying assumptions. Unfortunately, the validity of these assumptions is often inadequately discussed. Below, we discuss the identifying assumptions associated with several different decomposition methods, including the Beveridge-Nelson decomposition, the Hodrick-Prescott filter, Structural VAR decompositions, and the cointegration approach.

Perhaps the most widely used identifying assumption is that the trend component (or in the case of the output gap, potential output) follows a random walk. This assumption is equivalent to attributing all forecastable changes in level to the cyclic component. Such an assumption is the basis of the Beveridge-Nelson univariate decomposition, and is also used in Kuttner's (1992, 1994) state-space decomposition, and in Evans and Reichlin's (1994) multivariate Beveridge-Nelson decomposition.

How useful is such a decomposition for policy purposes?[8] That depends on whether the definition of the cyclic component is closely related to a useful economic interpretation of the output gap. For example, if we wish to calculate cyclically-adjusted budget deficits, then defining the gap to be the sum of predictable changes in output might be reasonable. However, if our aim is to use the output gap to estimate to estimate inflationary pressures, there is little reason to expect forecastable output growth *per se* to be particularly suitable. From a structural perspective, there is little reason to assume that potential output should follow a random walk (with or without drift). For example, it is easy to argue that "time-to-build" or the gradual diffusion of technology should generate important predictable changes in potential output.

Although the Hodrick Prescott filter[9] is frequently used to detrend data, the fact that its trend/cycle decomposition also has a specific identifying assumption is often overlooked. Given a time series $\{y_t\}$, the HP filter chooses its trend $\{\tau_t\}$ as the solution to

$$\min_{\{\tau_t\}} \sum_{t=1}^{T} (\tau_t - y_t)^2 + \lambda \cdot \sum_{t=2}^{T-1} (\Delta\tau_t - \Delta\tau_{t-1})^2 \qquad \text{(EQ 2)}$$

where λ is an exogenously given parameter. This can be rationalized as the solution to a signal-extraction problem where

$$y_t = \tau_t + v_t \qquad \text{(EQ 3)}$$

$$\Delta^2\tau_t = u_t \qquad \text{(EQ 4)}$$

where u_t, v_t are mutually uncorrelated normal i.i.d. variables and λ is a function of their relative variances.[10]

How useful is such a decomposition for policy purposes? Again, that depends on how rea-

8. Evans and Reichlin (1994) note that structural interpretations of such a forecast-based representation of trend and cycle are not warranted. (p. 242, Remark 2.) However, they also state (p. 234) that "One can thus interpret C_t as the gap between current and 'normal' output levels, and the size of this gap is clearly crucial for policy making."

9. In the preceding section, it was noted that the Hodrick Prescott filter could properly be called a smoother rather than a filter. In addition, it was not created by Hodrick and Prescott. Diewert and Wales (1993) note that the technique originated in the statistics literature in the 1920s and is more commonly known as a smoothing spline.

10. A number of extensions to the basic Hodrick Prescott filter have been investigated. The pioneering work in this area was Laxton and Tetlow (1992), who add the squared residuals from structural relationships to the minimization problem in (EQ 2). Coté and Hostland (1994) further generalize the problem by estimating the weighting parameters λ simultaneously with τ_t. Giorno, et al. (1995) combine an HP filter with a structural model. They all rest upon identifying assumptions analogous to those in the simple univariate HP filter.

sonable these identifying assumptions are. While the Beveridge-Nelson decomposition assumed that the expected growth rate of potential was a constant, the Hodrick-Prescott decomposition assumes that the expected growth rate of potential is a random walk and that the output gap is serially uncorrelated. Among other things, this would imply that real output growth should contain a unit root, a hypothesis which is usually easy to reject. It is therefore hard to argue that this is a plausible data generating process for output, and difficult to understand why output gaps generated by this method should be useful for policy.

A structural vector-autoregression model (SVAR) can refer to many different things. Both Sims (1986) and Bernanke (1986) made important early contributions to the literature on giving structural interpretations to reduced-form VAR models. However, in the context of estimating potential output, most of the attention now focuses on the approach pioneered by Blanchard and Quah (1989). Their key innovation was to use only long-run and orthogonality restrictions to recover structural shocks from the reduced form. Since macro-economic theory is much more specific about long-run behaviour than short-run dynamics, this was a very attractive approach that has been widely implemented and extended.[11]

Unfortunately, the orthogonality assumptions of such SVARs can pose problems for accurate measurement of the output gap. Specifically, the assumption that shocks are orthogonal at all leads and lags implies that the monetary policy reaction to a supply shock would be counted as part of the supply shock, even if this monetary response had no effect on potential output and solely affected the output gap. Also problematic may be the assumption that any shocks having only transitory effects on output are demand shocks. This means that transitory shocks to potential output would not be counted as supply shocks.[12] DeSerres, Guay and St. Amant (1994) consider the same problem from a slightly different perspective. They note that using only the effects of demand shocks to construct the output gap implicitly assumes that supply shocks have identical effects (even in the short-run) on potential and actual output. They explore the conditions under which this is true and conclude that the results will be sensitive to the policy rule. Consequently, we cannot be sure that the SVAR measure of the output gap will reflect the “true” gap.

Yet another approach to decomposing output movements into trend and cycle may be associated with cointegrating relationships. Suppose we find some set of variables $\{X_t\}$ that captures all the long-run movements in $\{Y_t\}$ (i.e. X_t and Y_t are cointegrated.)[13] One can therefore define the trend component of y_t to be x_t, $\tau_t = x_t \cdot \beta$ where β is the cointegrating vector. The stationary residual term defines the cyclic component.

11. Although the number of related papers is very large, the most important of these are the extension by King, Plosser, Stock and Watson (1991) to cointegrated systems, and the problem of non-fundamental solutions discussed by Lippi and Reichlin (1993, 1994). Applications to the measure of the output gap include DeSerres, Guay and St. Amant (1995).

12. Examples of transitory shocks to potential output are labour disputes, unusual weather and seasonal fluctuations. DeSerres, Guay and St. Amant (1994) give a good discussion of the importance of these problems and others.

13. There are a number of theoretical justifications for such relationships. For example, if we assume that Y_t is produced from a two-factor Cobb-Douglas production function $f(K_t, L_t)$, then the logs of Y_t, K_t, L_t should cointegrate. Alternatively, a forward-looking rational model of consumption behaviour can imply that the logs of C_t and Y_t are cointegrated.

The usefulness of such a decomposition once again depends on how reasonable these identifying assumptions are. In this case, although x_t cointegrates with y_t, we know that any stationary transformation of x_t will also cointegrate with y_t. We therefore have to ask why, of all these possible variables, we should rely on x_t to define the cyclic component. It might be argued that x_t captures all expected future changes in y_t, but we have already discussed (in the context of the random walk assumption) why such trend/cycle decomposition may be misleading. Similarly, it might be argued that $x_{t'}$ captures the long-run determinants of y_t, but we have already mentioned why short-run variation may be an important feature of potential output.

To summarize, there is an important lesson to be learned from the trend/cycle decomposition literature. If we wish to identify the cyclic component of output, we must have some way of choosing from an infinite number of different definitions. If we also want this cyclic component to have an economic interpretation that will be of use to policy makers, we must ensure that the definition which we choose is based on sound economic principles. The argument that “capturing the long-run trend in output” will be sufficient is vacuous. Imposing arbitrary assumptions on the dynamics of the trend component will lead to arbitrary definitions of the cyclic component. None of the methods surveyed manages to identify the cyclic component in an way that will necessarily be meaningful to a policy maker, although the random walk assumption may be justified in some circumstances.

IV. A Trivial Filter

This section introduces a new method for estimating the output gap which is designed to produce estimates that may be a useful guide to policy. To do so, it responds to several of the points raised in the preceding two sections. First, it follows in the spirit of Blanchard and Quah (1989), Kuttner (1992), Laxton and Tetlow (1992), Gregory and Smith (1994) by using economic theory to add structure to the problem and thereby assist in identifying the output gap. Second, wherever possible, it avoids imposing arbitrary assumptions on the dynamics of either the output gap or potential output or their interaction. Third, the method can be applied as a filter or as a smoother. The method uses a simple linear filter with some modest optimality properties. It will be referred to as TOFU - a Trivial Optimal Filter that's Understandable.[14]

We begin by considering a linear filter of actual output. In other words, assume that potential output can be expressed as

$$q_t = A(L) \cdot y_t + \varepsilon_t \tag{EQ 5}$$

where q_t is (the log of) potential output, y_t is (the log of) actual output, ε_t is an innovations process that is uncorrelated with y_t at all leads and lags, and $A(L)$ is a two-sided polynomial in the lag operator (i.e. it takes a weighted sum of leads, lags and contemporaneous values of y_t.) A sufficient but not necessary condition for such a representation to exist is that output y_t has a unit root and that the output gap $q_t - y_t$ is stationary.

We typically think of y_t as being non-stationary in mean, since it tends to drift upwards over time. To ensure that q_t and y_t move together in the long run (so that the gap has a stationary mean), we will further assume that

$$A(1) = 1 \tag{EQ 6}$$

which simply means that the weights in $A(L)$ must sum to one. This in turn implies that we can write

$$A(L) - 1 = (1 - L) \cdot \tilde{A}(L) \tag{EQ 7}$$

and therefore that

$$q_t - y_t = \tilde{A}(L) \cdot \Delta y_t + \varepsilon_t \tag{EQ 8}$$

Therefore, so long as (EQ 5) and (EQ 6) hold, we should be able to estimate the output gap as the weighted sum of past, present and future output growth.

From (EQ 8), we see that we can estimate the output gap in terms of the observable variable Δy_t if we can identify $\tilde{A}(L)$. Presumably, if we know of an economic relationship that involves the output gap, we could use this to define an optimal estimate of $\tilde{A}(L)$, call it $\hat{A}(L)$. For example, we might consider a Phillips Curve of the form

14. This filter was derived independently by the author. However, it is unlikely that anything so trivial would be original. The author would be grateful for references to earlier derivations and applications of such a filter, which will be cited in future drafts.

$$\pi_t = \alpha_0 + \gamma \cdot (q_t - y_t) + B(L) \cdot \pi_{t-1} + C(L) \cdot z_t + e_t \tag{EQ 9}$$

where z_t is a vector of additional observable variables, e_t is an i.i.d. mean zero error term and $B(L)$, $C(L)$ are one-sided polynomials in non-negative powers of L. We could substitute (EQ 8) into (EQ 9) to get

$$\pi_t = \alpha_0 + \gamma \cdot \left(\tilde{A}(L) \cdot \Delta y_t \right) + B(L) \cdot \pi_{t-1} + C(L) \cdot z_t + \tilde{e}_t \tag{EQ 10}$$

where $\tilde{e}_t = e_t + \gamma \cdot \varepsilon_t$. (EQ 10) can now be estimated by conventional methods to obtain optimal estimates of $\tilde{A}(L)$, since it is now specified entirely in terms of observable variables. This would allow us to estimate $\hat{A}(L) \cdot \Delta y_t$ and thereby use (EQ 8) to estimate the output gap.

Before proceeding, we should note two minor caveats. First, (EQ 10) identifies $\tilde{A}(L) \cdot \Delta y_t$ only up to the scaling factor γ. Strictly speaking, therefore, we only recover an index of the output gap. We assume that this is sufficient to guide policy. This should be reasonable for the purpose of, say, deciding whether interest rates should be higher or lower to achieve a given target, since the current value of the index can be readily compared to its historical values. Of course, for some applications (such as calculating the output cost of reducing inflation) an index of the output gap will not be sufficient.

Second, consistent estimation of this relationship requires an implicit assumption. OLS estimation requires $cov(e_t, \Delta y_{t-j}) = 0 \quad \forall j$ for consistency. If this condition is not satisfied, then instruments for Δy will be required for estimation. Consistent IV estimation in turn will depend on the assumption that the chosen instruments are valid.

Now consider the properties of this estimator. Estimation by maximum likelihood is straightforward, so our estimates of $\tilde{A}(L)$ will be efficient, giving us the modest optimality properties referred to earlier. The estimator imposes only the most general assumptions on the time series properties of the series involved. It incorporates a simple structural relationship in order to identify the output gap. This estimator therefore avoids some of the most serious problem associated with the techniques discussed in the preceding sections.

It may also be useful to test whether the level of the output gap is significantly positive or negative at some point in time. From (EQ 8), this is equivalent to a one-sided test of $\hat{A}(L) \cdot \Delta y_t + \varepsilon_t = 0$. Unfortunately, an exact test of this hypothesis is not possible without more information on the distribution of ε_t. However, a test of $\hat{A}(L) \cdot \Delta y_t = 0$ is straightforward and can be done using standard inference methods. Provided that the variance of ε_t is not too large, this may serve as a useful upper bound on the certainty with which we can estimate the output gap.

A final desirable property would be to produce both a filtered and a smoothed estimate of the output gap. Since $\tilde{A}(L)$ may be two-sided, the estimator defined above will generally produce a smoothed estimate. In the next section, we examine the implications of constraining $\tilde{A}(L)$ to be one-sided, so that it produces a filtered estimate of the output gap.

Before moving to the next section however, note that the TOFU estimate of the output gap can be extended in a number of ways. If the output gap were to enter the structural equation in a non-linear fashion, we could estimate the system via GMM rather than least-squares techniques.

If we had a series of structural equations involving the output gap, we could estimate them simultaneously subject to cross-equation restriction on the coefficients of Δy_t. If we wished to combine both time-series and structural estimates of the output gap, we could include both $\tilde{A}(L) \cdot y_t$ and structural estimates of the output gap ψ_t in (EQ 10).

V. Filtering and Causality

Suppose that we wish to use the TOFU method to estimate the current value of the output gap. However, we presumably have no information on present or future values of Δy_t.[15] In that case, our optimal estimate $\hat{Q}(L) \cdot \Delta y_t$ would impose the restriction that only lagged values of Δy_t enter (EQ 10). To understand how this will affect the accuracy of our estimate, note that

$$E(q_t - y_t | H_y) = \tilde{A}(L) \cdot \Delta y_t \text{ and} \quad \text{(EQ 11)}$$

$$V((q_t - y_t) - E(q_t - y_t | H_y)) = \sigma^2 \quad \text{(EQ 12)}$$

where H_y is the set of all past, present and future values of y_t. By the law of iterated expectations and (EQ 11)

$$E(q_t - y_t | H_y^-) = E(E(q_t - y_t | H_y) | H_y^-) = E(\tilde{A}(L) \cdot \Delta y_t | H_y^-) \quad \text{(EQ 13)}$$

where H_y^- is the set of all past values of y_t. If we define

$$\tilde{A}(L) = \tilde{A}^-(L) + \tilde{A}^+(L) \quad \text{(EQ 14)}$$

where $\tilde{A}^-(L)$ has only positive powers of L and $\tilde{A}^+(L)$ has only non-positive powers of L, then (EQ 13) implies

$$E(q_t - y_t | H_y^-) = E(\tilde{A}^-(L) \cdot \Delta y_t | H_y^-) + E(\tilde{A}^+(L) \cdot \Delta y_t | H_y^-) = \tilde{A}^-(L) \cdot \Delta y_t + \sum_{j=1}^{m} a_j^+ \cdot E(\Delta y_{t+j} | H_y^-) \quad \text{(EQ 15)}$$

where a_j^+ is simply the coefficient on L^{-j} in $\tilde{A}^+(L)$. Similarly, we can show that

$$V(q_t - y_t | H_y^-) = V(q_t - y_t | H_y) + V(\tilde{A}(L) \cdot \Delta y_t | H_y^-) = \sigma^2 + V(\tilde{A}^+(L) \cdot \Delta y_t | H_y^-) \quad \text{(EQ 16)}$$

where $V(X|\Omega)$ is the variance of the error in forecasting X given the information set Ω.

(EQ 15) and (EQ 16) have an intuitive interpretation. The extent to which $\hat{Q}(L) \cdot \Delta y_t$ is less informative than $\hat{A}(L) \cdot \Delta y_t$ will depend on the weight which $\tilde{A}(L)$ puts on current and future values of Δy_t and the extent to which those future values can be predicted from current and past values. The former will in turn depend on the Granger causal relationship between $q_t - y_t$ and Δy_t, while the latter will depend on the degree to which output growth is serially correlated.

Table 1 shows the autocorrelations for real GDP growth for various countries, frequencies, time periods and measures. Table 2 uses the same data and shows the R^2 from a regression of real GDP growth on its own lags. For the industrialized nations, 12 lags of quarterly output growth predicts only 20-40% of the variance of current output growth. Much of this explanatory power

15. We'll assume that current Δy_t is not available due to reporting lags. Relaxing this assumption does not substantially alter the results.

seems to come from the first few lags. These results suggest that since predictability can be low, the extent of Granger-causality may play an important role in determining how accurately the TOFU filter can estimate current output.

Now let's consider the effect of the Granger-causal relationship between $q_t - y_t$ and Δy_t. Suppose that Δy_t does not Granger cause $q_t - y_t$. This implies that

$$\tilde{A}(L) = \tilde{A}^{+}(L) \qquad \text{(EQ 17)}$$

so only current and future values of output growth can give us any information about the output gap. As we can see from (EQ 15)and (EQ 16), this is almost precisely the case where $\hat{Q}(L) \cdot \Delta y_t$ is as uninformative as possible. We will therefore be unable to estimate the current output gap with much precision unless we can accurately forecast output growth. However, the results above suggest that the growth of output is itself difficult to predict. **We can therefore conclude that the question of whether (or more accurately, to what extent) actual output growth Granger causes the output gap will be important for the accuracy that we could hope to achieve in measuring the output gap with this kind of filtering approach.**

At this point, a brief digression is needed to consider whether applying the concept of Granger-causality to an unobserved series like the output gap or potential output makes sense. It does, provided that one correctly understands the nature of the thought experiment involved. When we talk about Granger causality between $q_t - y_t$ and Δy_t, this makes sense only in a context where both can be observed.[16] When we study dynamic macroeconomic models (including large-scale models such as INTERMOD, DRI, or QPM) we have variables representing both $q_t - y_t$ and Δy_t, so their interrelationship can be studied. These models therefore have *conceptual* implications for the Granger-causal relationship between $q_t - y_t$ and Δy_t. However, this does not imply that the implications of these models are directly testable on real world data. Our inability to observe $q_t - y_t$ prevents that. Nonetheless, our beliefs about the Granger-causal relationship between $q_t - y_t$ and Δy_t have implications for how we *should* be able to measure the output gap.

So what kinds of models are consistent with using time series methods for measuring the current output gap? We can usefully discuss three cases; (1) the case where a TOFU-filter will tell us as much about the output gap as a TOFU-smoother, (2) the case where a TOFU-filter will tell us less about the output gap than a TOFU-smoother, and (3) the case where a TOFU-filter will give us no information about the output gap. For simplicity, throughout this discussion, we will assume that the past history of output growth is of no use in predicting present and future output growth.

From (EQ 15) we can see that the TOFU-filter will tell us as much about the output gap as a TOFU-smoother when $\tilde{A}^{+}(L) = 0$, which in turn implies that $q_t - y_t$ does not Granger-cause

16. For example, we say that X does not Granger cause Y iff

$$P\left[Y_t \middle| H_X^-, H_Y^-\right] = P\left[Y_t \middle| H_Y^-\right] \qquad \text{(EQ 1)}$$

Confusion arises if H_X^- is the empty set, since this would imply that (EQ 1) must always be satisfied. Obviously, Granger causality will only be of interest for some non-empty set H_X^-.

Δy_t. In the appendix, it is shown that the latter condition in turn implies that q_t does not Granger-cause y_t. If y_t and q_t are cointegrated, this would imply that there is unidirectional causality from y_t to q_t. In other words, exogenous shocks to potential output would have no subsequent effect on actual output, but persistent shocks to actual output would eventually appear to cause a similar change in potential. Such behaviour could describe a particularly severe form of hysteresis; one where output has no tendency to return to potential and instead potential output is driven in the long run only by previous variations in actual output. In this kind of world, time series methods can hope to be as effective in estimating the current output gap as they are in estimating past output gaps.

The latter will not be the case when $q_t - y_t$ Granger-causes Δy_t, or equivalently when q_t Granger-causes y_t. Therefore, so long as actual output appears to respond to some degree to past changes in potential output, then time-series methods will estimate the current output gap less accurately than past output gaps. The intuition behind this result should be clear. Since future output growth will reflect the influence of the current output gap, we can gain information about the current gap by observing future growth. Note that Granger-causality from q_t to y_t does not rule out additional Granger-causality from y_t to q_t. Therefore, this case does not rule out the presence of *some* hysteresis-like effects of the kind described above. It simply implies that the latter cannot be the *only* force linking actual and potential output.

Time series methods will be of no use in estimating the current output gap when Δy_t does not Granger cause $q_t - y_t$.[17] In other words, if faster or slower than normal output growth tends to have no subsequent effect on the size of the output gap, then time series estimates of the current gap will be as uninformative as possible. The intuition is similar to that above. Past output growth is the only information about the gap that we have; if it tells us nothing about the current gap, then our estimates will be unillumanating. It is more difficult to characterize the kind of economic model which will generate this kind of result since Granger-causality from Δy_t to $q_t - y_t$ does directly correspond to any statement about Granger-causality between y_t and q_t.[18] However, it is possible to give examples in which this result would hold.

Consider the case where

$$\begin{aligned} \Delta y_t &= \alpha \cdot (y_{t-1} - q_{t-1}) + u_t \\ q_t &= q_{t-1} + v_t \end{aligned} \qquad \text{(EQ 18)}$$

Potential output follows a random walk that is independent of the behaviour of output. Actual output in turn is generated by a simple error-correction model, which ensures that actual and potential output move together in the long run. Such a model precisely satisfies the condition for no Granger causality from Δy_t to $q_t - y_t$. More generally, we can consider the class of models where there is no Granger causality from y_t to q_t as

17. Again, this conclusion assumes that past output growth is of no use in predicting future variations in output growth. As was mentioned earlier, the data show that there is some slight serial correlation in output growth, so time series methods would still have some slight explanatory power even in this case.

18. See Appendix.

$$y_t = \sum_{j=1}^{p} \alpha_j \cdot y_{t-j} + \sum_{j=1}^{p} \beta_j \cdot q_{t-j} + u_t$$

$$q_t = \sum_{j=1}^{p} \delta_j \cdot q_{t-j} + v_t \qquad \text{(EQ 19)}$$

An absence of Granger causality from Δy_t to $q_t - y_t$ requires

$$\delta_j = \beta_j + \alpha_j \qquad \forall j \qquad \text{(EQ 20)}$$

and will therefore imply an inability of time series methods to estimate the current output gap. In models where there is no Granger causality from q_t to y_t, which may be written as

$$y_t = \sum_{j=1}^{p} \alpha_j \cdot y_{t-j} + u_t$$

$$q_t = \sum_{j=1}^{p} \delta_j \cdot q_{t-j} + \sum_{j=1}^{p} \gamma_j \cdot y_{t-j} + v_t \qquad \text{(EQ 21)}$$

the equivalent condition is

$$\delta_j + \gamma_j = \alpha_j \qquad \forall j \qquad \text{(EQ 22)}$$

Clearly, there is a range of models in which time-series methods will be of little use in estimating current output gaps. Furthermore, it is the short-run dynamics of potential and actual output which are critical in determining whether models belong to this class. Since economic theory will generally have little to say about such short-run behaviour, the usefulness of time-series filters will depend largely on empirical rather than theoretical questions.

It is well beyond the scope of this paper to argue which of the many economic characterizations mentioned above is most appropriate. However, it appears that a filtered estimate of the output gap will be as accurate as a smoothed estimate when actual output does not adjust to past changes in potential. Furthermore, regardless of the causal relationship between q_t and y_t, there is no guarantee that a filtered estimate can tell us *anything* about the current output gap.

VI. Conclusions

In order for time series estimates of the output gap to be useful to policy makers, this paper has argued that two factors will be critical. First, they must be able to produce an estimate of the *current* output gap based only on past information. Put another way, to evaluate the performance of such estimators, we should focus on their properties as *filters* rather than *smoothers*. Second, the decomposition of actual output into potential output and the output gap must not be based on arbitrary assumptions on the time-series behaviour of these variables. In general, this leads to a multiplicity of possible solutions that may differ greatly in their policy implications. The arbitrary selection of one of these creates arbitrary policy advice.

A trivial but flexible estimator of the output gap (dubbed TOFU) can be constructed which satisfies both of the above criteria. It estimates the output gap as part of a larger economic relationship. A discussion of its properties shows that except under very strict circumstances, such time series methods will produce better estimates of output gaps *ex post* than they will *ex ante*. Furthermore, under certain conditions, contemporary and historical data will be of no use in estimating the output gap whatsoever. This in turn places limits on the extent to which time series methods can hope to improve upon structural estimates of the output gap.

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VIII. Appendix - Some Granger-Causality Results

It is useful to understand the relationships in the Granger-causal behaviour of two pairs of variables; $\{y_t, q_t\}$ and $\{\Delta y_t, q_t - y_t\}$. First, note that we can write

$$\begin{bmatrix} \Delta y_t \\ q_t - y_t \end{bmatrix} = \begin{bmatrix} 1-L & 0 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_t \\ q_t \end{bmatrix} = P \cdot \begin{bmatrix} y_t \\ q_t \end{bmatrix} \qquad \text{(EQ 23)}$$

Next, note that

$$P^{-1} = \begin{bmatrix} (1-L)^{-1} & 0 \\ (1-L)^{-1} & 1 \end{bmatrix} \qquad \text{(EQ 24)}$$

Now, suppose that $\{u_t, v_t\}$ are innovation sequences and that

$$\begin{bmatrix} y_t \\ q_t \end{bmatrix} = \begin{bmatrix} \alpha(L) & \beta(L) \\ \gamma(L) & \delta(L) \end{bmatrix} \cdot \begin{bmatrix} y_t \\ q_t \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \Lambda \cdot \begin{bmatrix} y_t \\ q_t \end{bmatrix} + U_t \qquad \text{(EQ 25)}$$

(EQ 23) and (EQ 25) imply that

$$\begin{bmatrix} \Delta y_t \\ q_t - y_t \end{bmatrix} = P \cdot \Lambda \cdot P^{-1} \cdot \begin{bmatrix} \Delta y_t \\ q_t - y_t \end{bmatrix} + P \cdot U_t \qquad \text{(EQ 26)}$$

From the definitions of P and Λ, we can show that

$$P \cdot \Lambda \cdot P^{-1} = \begin{bmatrix} \alpha(L) + \beta(L) & (1-L) \cdot \beta(L) \\ (1-L)^{-1} \cdot (\gamma(L) + \delta(L) - \beta(L) - \alpha(L)) & \delta(L) - \beta(L) \end{bmatrix} \qquad \text{(EQ 27)}$$

We know that y_t Granger-causes q_t iff $\gamma(L) \neq 0$ and that q_t Granger-causes y_t iff $\beta(L) \neq 0$. Similarly, from (EQ 26) and (EQ 27) we can say that y_t Granger-causes $q_t - y_t$ iff $\gamma(L) + \delta(L) - \beta(L) - \alpha(L) \neq 0$ and that $q_t - y_t$ Granger-causes Δy_t iff $\beta \neq 0$. We can therefore conclude that $q_t - y_t$ Granger-causes Δy_t iff q_t Granger-causes y_t.

Table 1: Autocorrelations of Real GDP

# Lags	Canada	Canada	Canada	USA	OECD Europe	France	Italy	Mexico	New Zealand	Total OECD	UK	UK	Argentina	Brazil	Chile	Venezuala
1	0.358	-0.078	0.470	0.370	0.243	0.259	0.295	-0.844	-0.441	0.484	0.016	-0.008	-0.089	0.208	-0.058	0.280
2	0.183	0.147	0.248	0.218	0.219	0.280	0.184	0.828	-0.056	0.352	0.052	0.075	0.023	0.250	-0.248	0.096
3	0.135	0.213	0.158	0.039	0.270	0.172	-0.021	-0.807	0.137	0.280	0.057	0.181	-0.086	0.140	0.022	0.047
4	0.084	0.098	0.006	-0.047	0.177	0.022	-0.164	0.807	-0.163	0.161	0.152	-0.006	0.078	0.149	0.561	0.181
5	0.062	0.083	0.038	-0.063	0.083	0.117	-0.354	-0.777	0.107	0.069	-0.002	-0.013	-0.111	0.004	-0.260	0.098
6	-0.154	0.069	-0.049	-0.012	0.127	-0.038	-0.225	0.704	0.011	0.133	-0.021	0.145	0.021	0.188	-0.365	0.113
7	0.133	0.088	-0.010	-0.042	0.087	0.075	-0.063	-0.728	0.113	0.080	-0.153	-0.066	-0.088	0.178	-0.061	-0.207
8	0.228	0.072	-0.011	-0.105	0.022	-0.098	-0.127	0.707	-0.208	-0.059	0.008	-0.166	0.073	0.130	0.463	-0.128
9	0.083	0.068	-0.009	-0.041	0.167	0.119	0.121	-0.698	0.062	0.106	-0.100	-0.023	-0.065	0.098	-0.221	0.007
10	0.120	0.015	0.004	0.059	0.043	0.171	0.267	0.640	0.121	0.157	-0.089	-0.022	0.020	-0.145	-0.243	-0.025
11	0.100	0.110	-0.152	-0.003	0.176	0.080	0.200	-0.656	-0.114	0.078	-0.133	-0.099	-0.051	0.103	0.061	0.035
12	0.124	-0.086	-0.113	-0.144	0.121	-0.020	0.167	0.654	0.089	0.031	-0.019	-0.113	0.070	-0.146	0.473	-0.015
Start:	1962	61M2	76Q2	47Q2	60Q2	70Q2	70Q2	80Q2	82Q3	60Q2	48Q2	55Q2	68Q2	1964	80Q2	1958
End:	1993	94M7	94Q2	94Q2	93Q4	94Q2	90Q3	94Q2	94Q1	93Q4	94Q2	94Q2	90Q4	1992	94Q1	1993
Series	I34026	I37026	D20883	Q.GDP87$	EUR2	FR2	HIT2	MX2	NZ2	OECD2	UK1	UK19	ARG2	BR2	CHI2	VENZ2

Note: The series name shown is that taken from ETS.

Table 2: R^2 from an Auto Regression - Real GDP

# Lags	Canada	Canada	Canada	USA	OECD Europe	France	Italy	Mexico	New Zealand	Total OECD	UK	UK	Argen-tina	Brazil	Chile	Venez-uala
1	0.0020	0.0365	0.2819	0.1693	0.0652	0.0435	0.2598	0.7931	0.1617	0.2527	0.0019	0.0000	0.2336	0.7350	0.0299	0.6909
2	0.0159	0.0446	0.2820	0.1746	0.0898	0.1001	0.2598	0.8159	0.2734	0.2642	0.0090	0.0064	0.2338	0.7366	0.4261	0.6936
3	0.0829	0.0447	0.2831	0.1798	0.1439	0.1030	0.3049	0.8435	0.2734	0.2735	0.0164	0.0413	0.4467	0.7384	0.5539	0.6968
4	0.0953	0.0463	0.2853	0.1804	0.1458	0.1398	0.3386	0.8577	0.2937	0.2742	0.0236	0.0414	0.5958	0.7407	0.6993	0.7008
5	0.0973	0.0467	0.2882	0.1814	0.1460	0.1398	0.3646	0.8578	0.3023	0.2758	0.0237	0.0430	0.6077	0.7751	0.7042	0.7178
6	0.0973	0.2072	0.3023	0.1830	0.1478	0.1483	0.3656	0.8628	0.3203	0.2896	0.0246	0.0552	0.6087	0.7754	0.7221	0.7191
7	0.0979	0.2266	0.3063	0.1852	0.1486	0.1575	0.3659	0.8676	0.3379	0.2898	0.0527	0.0589	0.6092	0.7757	0.7469	0.7205
8	0.0991	0.2936	0.3077	0.1957	0.1492	0.1775	0.3741	0.8716	0.3387	0.3159	0.0530	0.0940	0.6099	0.7760	0.7550	0.7221
9	0.0999	0.2987	0.3088	0.1968	0.1737	0.1912	0.3877	0.8731	0.3427	0.3592	0.0601	0.0980	0.6100	0.7858	0.7575	0.7325
10	0.1004	0.3381	0.3107	0.2099	0.1737	0.2141	0.4276	0.8749	0.3442	0.3641	0.0637	0.0989	0.6113	0.7860	0.7680	0.7333
11	0.1042	0.3965	0.3421	0.2145	0.1985	0.2144	0.4340	0.8824	0.3444	0.3658	0.0713	0.0998	0.6156	0.7862	0.7680	0.7341
12	0.1177	0.6408	0.3434	0.2542	0.1986	0.2625	0.4341	0.8889	0.3457	0.3720	0.0714	0.1186	0.6162	0.7864	0.7868	0.7350
Start:	1962	61M2	76Q2	47Q2	60Q2	70Q2	70Q2	80Q2	82Q3	60Q2	48Q2	55Q2	68Q2	1964	80Q2	1958
End:	1993	94M7	94Q2	94Q2	93Q4	94Q2	90Q3	94Q2	94Q1	93Q4	94Q2	94Q2	90Q4	1992	94Q1	1993
Series	I34026	I37026	D20883	Q.GDP87$	EUR2	FR2	HIT2	MX2	NZ2	OECD2	UK1	UK19	ARG2	BR2	CHI2	VENZ2

Note: Figures reported above are the R^2 from a regression of output on a constant and the indicated number of lags of itself. This table uses the same data as the previous table.